EXPLORING

PHYSICS

YEAR 12 - EXPERIMENTS, INVESTIGATIONS & PROBLEMS

Worked Solutions

The STAWA *Worked Solutions* have been developed through the collaboration of teachers working in Department of Education, Catholic Education WA and Association of Independent Schools of WA. Funding assistance was provided by the Department of Education.

The *Worked Solutions* are intended to support the problem sets of the STAWA ATAR Exploring Physics Year 12: experiments, investigations and problems.

In an endeavour to provide the highest quality publication, the STAWA *Worked Solutions* were written and checked by different teachers. This does not guarantee that all answers are correct. Teachers are advised to work through disputed solutions with their students. If they are sure there is an error then they are asked to forward corrections to STAWA by email: admin@stawa.net

The STAWA *Worked Solutions* are a great example of teachers helping teachers for the benefit of all students.

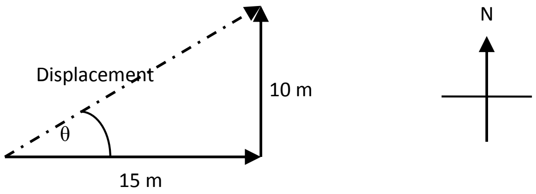
STAWA
SCIENCE TEACHERS' ASSOCIATION OF WESTERN AUSTRALIA

YEAR 12 ATAR

Problem Set 1: Vector additions, subtractions and resolution

1. Velocity, speed in a given direction, is a vector quantity and as such has both magnitude and direction.

2. a) Players displacement?

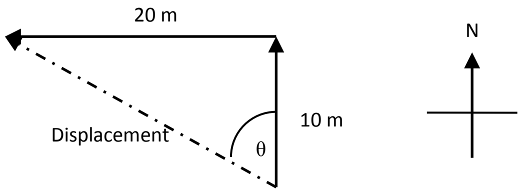


$s = \sqrt{(15^2 + 10^2)}$
$s = 18.0$ m

$\theta = \text{Arctan}\ (10 / 15)$
$\theta = 33.7^0$

$s = 18.0$ m East 33.7^0 North or **$s = 18.0$ m North 56.3^0 East.**

b) Ball displacement



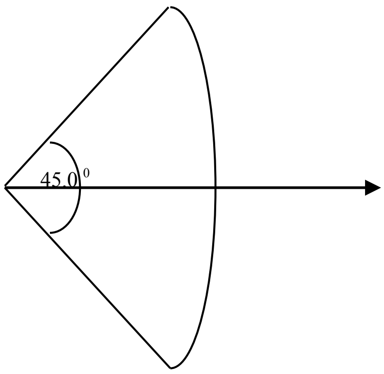
$s = \sqrt{(10^2 + 20^2)}$
$s = 22.4$ m

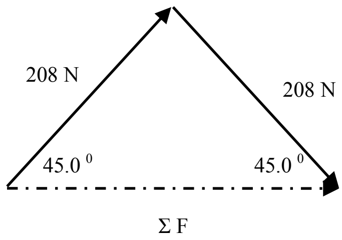
$\theta = \text{Arctan}\ (20 / 10)$
$\theta = 63.4^0$

$s = 22.4$ m North 63.4^0 West or **$s = 22.4$ m West 26.6^0 North**

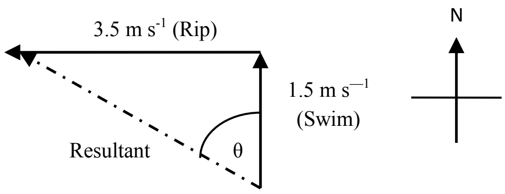
3. Force on arrow

$\Sigma F = \sqrt{(208^2 + 208^2)}$
$\Sigma F = 294$ N Forwards





4. Assuming the swimmer is swimming north …



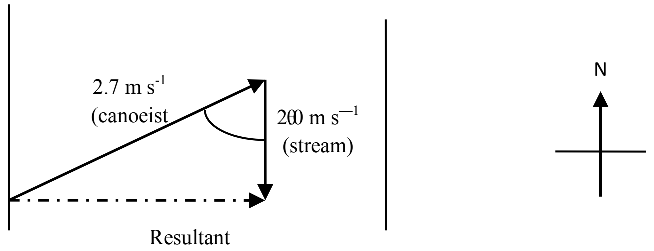
$v = \sqrt{(3.5^2 + 1.5^2)}$
$v = 3.81$ m s^{-1}

$\theta = \text{Arctan}\ (3.5 / 1.5)$
$\theta = 66.8^0$

$v = 3.81$ m s^{-1} North 66.8^0 West or **$v = 3.81$ m s^{-1} West 23.2^0 North.**

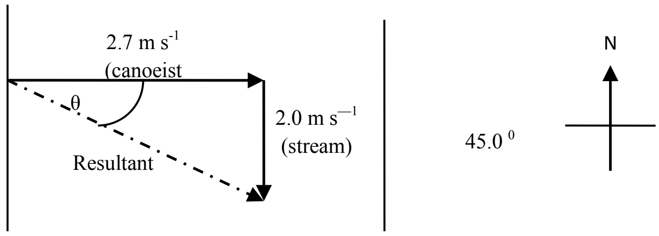
5. Canoeist

a) θ = Arcos (2.0 / 2.7)
θ = 42.2^0 to the current
(θ = 47.8^0 to the bank)



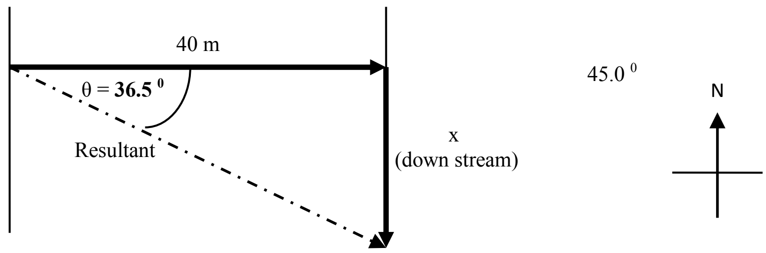
b) v = $\sqrt{(2.7^2 + 2.0^2)}$
v = 3.36 m s^{-1}

θ = Arctan (2.0 / 2.7)
θ = 36.5^0

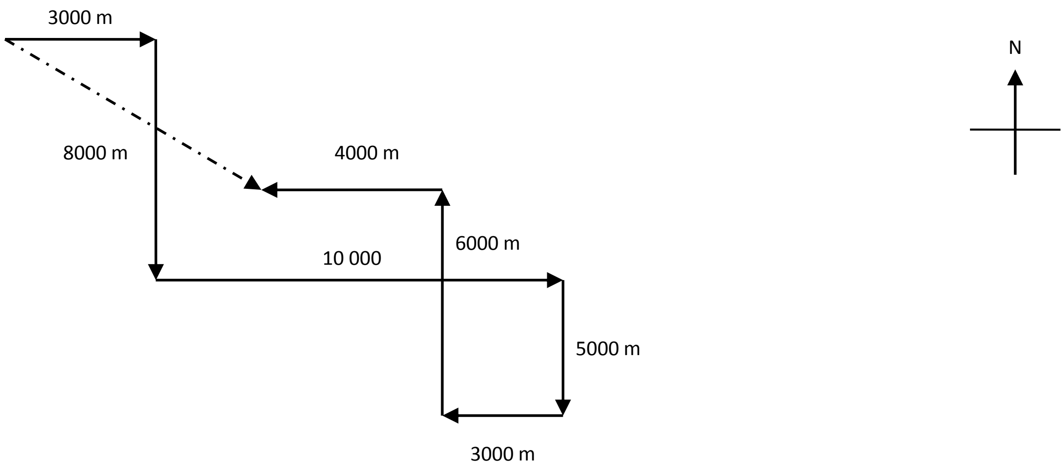


v = 3.36 m s^{-1} at 53.5^0 to the bank and angled downstream.

c)
Tan 36.5 = (x / 40)
x = 29.6 m down stream



6. a) Cross country skier: This diagram is **not** to scale.
An appropriate scale for a student to use would be 1 cm = 1 000 m



Add north – south and east - west vectors separately using a sign convention.

	North – South (+ / -)	**East – West (+ / -)**
	- 8000 m	+ 3000
	- 5000 m	+ 10 000
	+ 6000 m	- 3000
		-4000
Total:	**-7 000 m**	**+ 6 000 m**

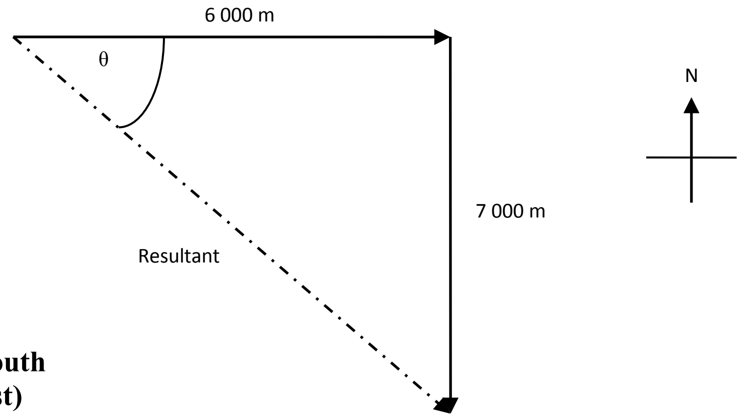
6. a) continued

$s = \sqrt{(6\,000^2 + 7\,000^2)}$

$s = 9.22 \times 10^3$ m

$\theta = \text{Arctan}\ (7\,000 / 6\,000)$

$\theta = 49.4^0$



$s = 9.22 \times 10^3$ m East 49.4^0 South

(9.22×10^3 m South 40.6^0 East)

b) Return journey is $s = 9.22 \times 10^3$ m North 40.6^0 W

7. Let towards the netballer be the positive direction and away negative.

$\Delta v = v - u$

$\Delta v = [0 - (+5)]$

$\Delta v = -5$ m s^{-1} or 5 m s^{-1} backwards

8. Let towards the tennis player be positive and away from the player be negative.

$u = +80 / 3.6 = +22.2$ m s^{-1}

$v = -90 / 3.6 = -25.0$ m s^{-1}

$\Delta v = v - u$

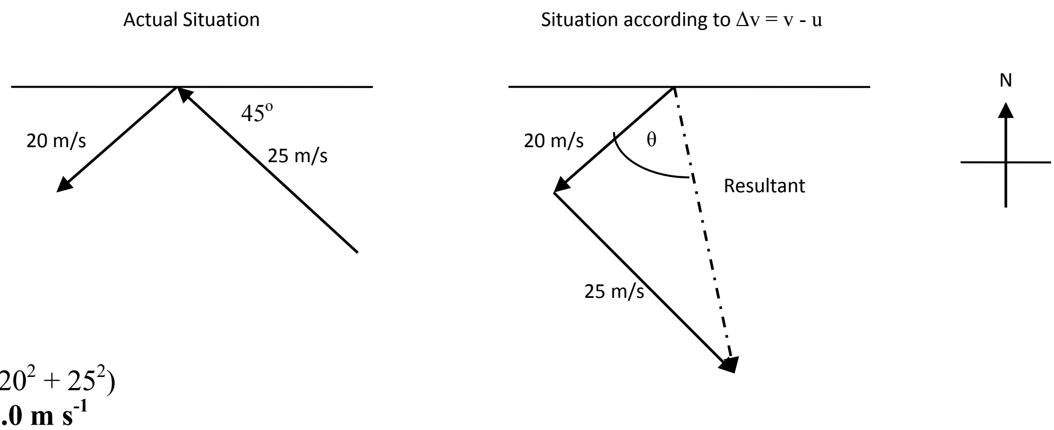
$\Delta v = [(-25) - (+22.2)]$

$\Delta v = [(-25) - 22.2]$

$\Delta v = -47.2$ m s^{-1} or 47.2 m s^{-1} away from the player.

Or -170 km h^{-1}

9.



$\Delta v = \sqrt{(20^2 + 25^2)}$

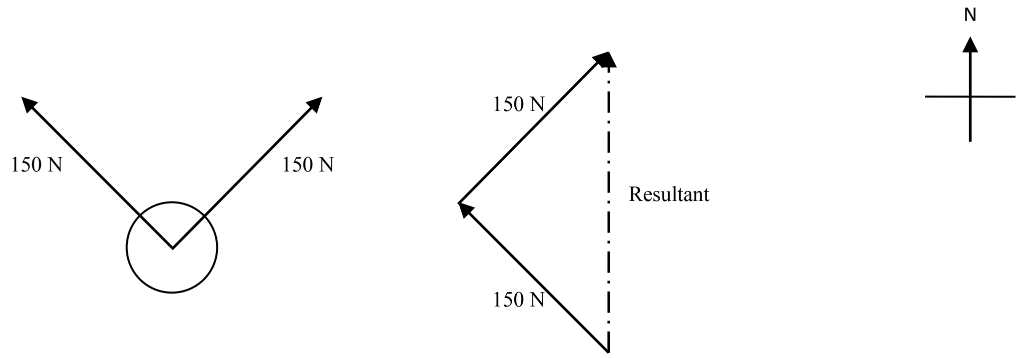
$\Delta v = 32.0$ m s^{-1}

$\theta = \text{Arctan}\ (25 / 20)$

$\theta = 51.3^0$

$\Delta v = 32.0$ m s^{-1} at $\theta = 51.3^0$ to the final velocity vector or at 96.3^0 to the wall.

10.



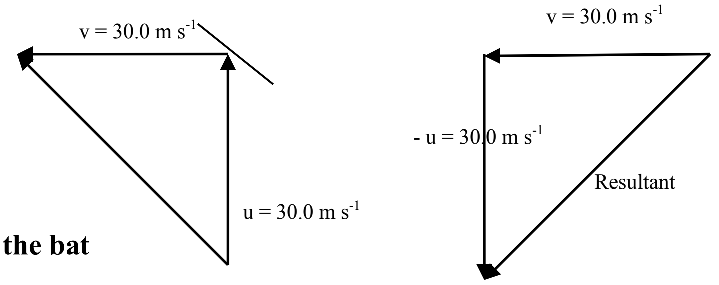
$F = \sqrt{(150^2 + 150^2)}$
$F = 212$ N

11.

$\Delta v = \sqrt{(30.0^2 + 30.0^2)}$
$\Delta v = 42.4$ m s^{-1}

θ = at right angles to the bat

$v = 42.4$ m s^{-1} at right angles to the bat



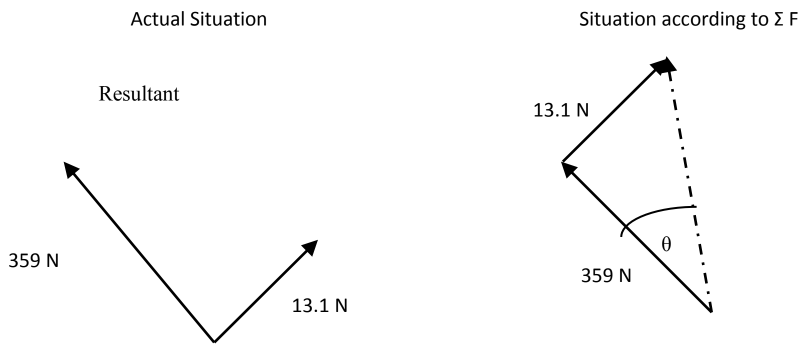
12.
Position 1
Towards the moon is positive. Towards the earth is negative.

$\Sigma F = (-480) + (+53.2)$
$\Sigma F = -426.8$ N
$\Sigma F = 426.8$ N towards Earth

Position 2

$\Sigma F = \sqrt{(359^2 + 13.1^2)}$
$\Sigma F =$ **359.2 N**

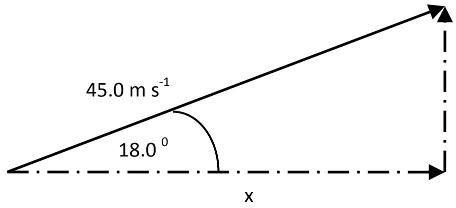
$\tan\theta = 13.1 / 359.$
$\theta = 2.09^0$



$\Sigma F =$ **359 N at 2.09^0 away from the line joining the asteroid to the earth. Angle bends towards the moon.**

13.

$\cos 18^0 = x / 45$
$x = 45 \cos 18^0$
$v = 42.8$ m s^{-1} horizontally

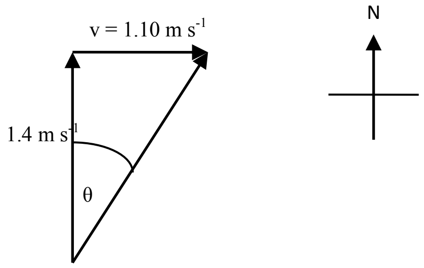


14.

$v = \sqrt{(1.4^2 + 1.1^2)}$
$v = 1.78$ m s^{-1}

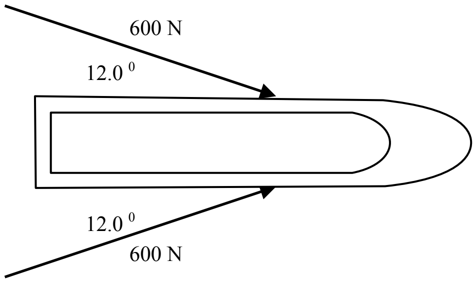
$\tan \theta = 1.1 / 1.4$
$\theta = 38.2^0$

$v = 1.78$ m s^{-1} North 38.2^0 East



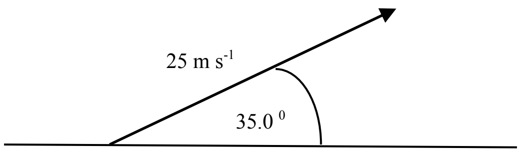
15. a)

$F = 2 \times (600 \cos 12)$
$F = 1173.8$ N
$F = 1.17 \times 10^3$ N Forward



b) $600 \sin 12 = 125$ N

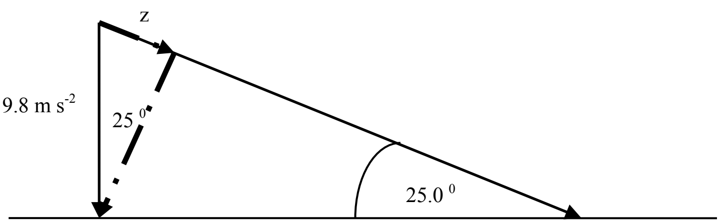
16. For rate is vague rate of change of displacement (velocity)



(ignoring air resistance)	**Vertical**	**Horizontal towards the pin**
Velocity	Initially = 25 sin (35) = 14.3 m/s and then decreasing due to gravitational acceleration.	25 cos (35) = 20.5 m/s and holding constant.

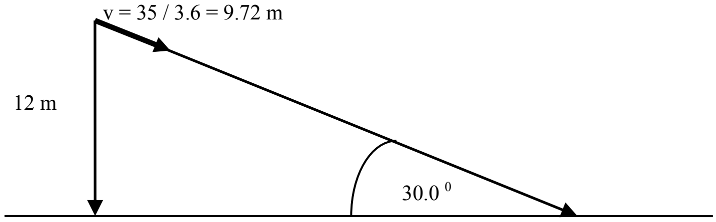
17.

sin 25 = z / 9.8
z = 4.14 m s^{-2}
a down the slope = 4.14 m s^{-2}



18.
Distance down the slope = ?
s = 12 / sin 30
s = 24 m down the slope

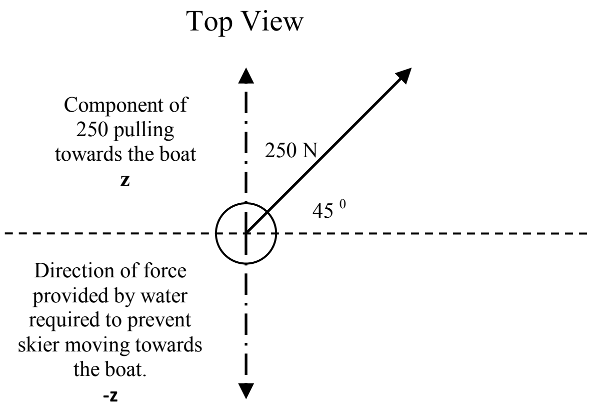
v = s / t
t = s / v
t = 24 / 9.72
t = 2.47 s



19.

Force pulling towards a boat
z = 25 sin 45
z = 17.7 N towards the boat

-z = 17.7 N = 17.7 N away from the boat



20. a)

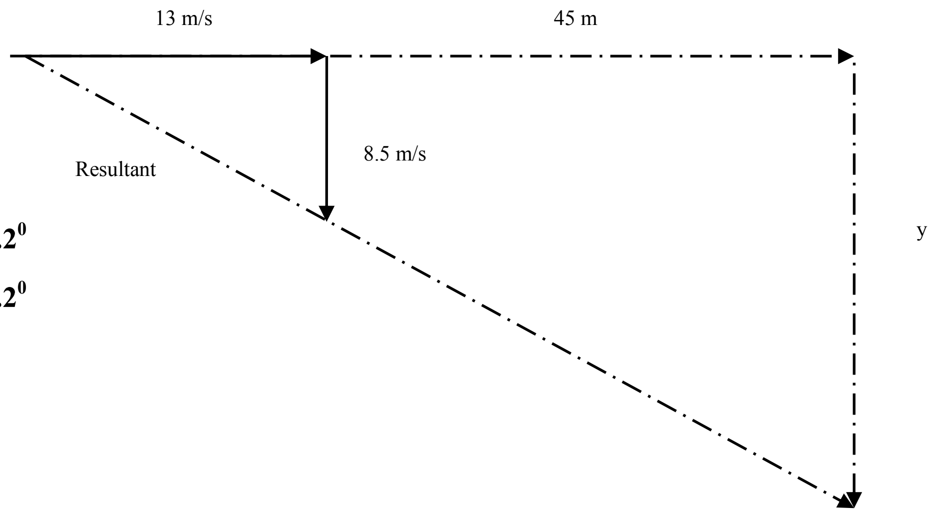
v = $\sqrt{(13^2 + 8.5^2)}$
v = 15.5 m s^{-1}

Tan θ = 8.5 / 13
θ = 33.2^0

$s_{hypotenuse}$ = 45 / cos 33.2^0

$s_{hypotenuse}$ = 45 / cos 33.2^0
$s_{hypotenuse}$ = 53.77 m

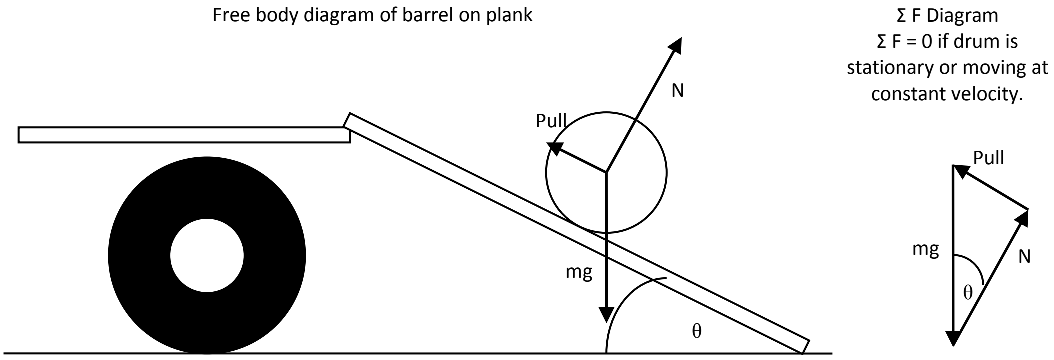
v = s / t
t = s / v
t = 53.77 / 15.5
t = 3.47 s



b)
tan (33.2) = y / 45
y = 45 tan 33.2
y = 29.5 m

21. a) The plank could be set at an angle to reduce the normal force that is required.

b) When the normal force on the plank = 166 kg x 9.8 = 1626.8 N the plank breaks

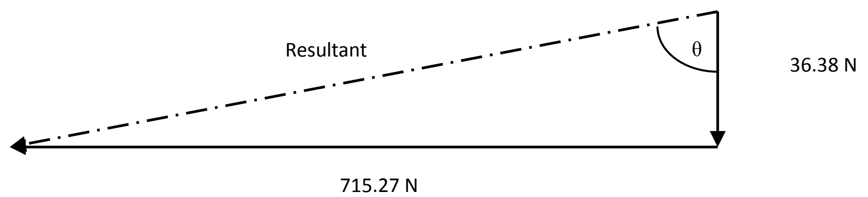


Drum Weight = 197 x 9.8 = 1930.6 N

Cos θ = N / mg
θ = Arc cos (1626.8 / 1930.6)
θ = 32.6^0

22. Skiers behind boat. Force on boat = ?

	Y axis (+/-)	**X axis (+/-)**
220 at 30^0 above	220 sin 30 = 110 N	-220 cos 30 = -190.52 N
180 at 12^0 above	180 sin 12 = 37.42 N	-180 cos12 = -176.07 N
170 at 10^0 below	-170 sin 10 = -29.52 N	-170 cos10 = -167.42 N
200 at 25^0 below	-200 sin 25 = - 84.52	-200 cos25 = -181.26 N
Total	+36.38 N	-715.27 N

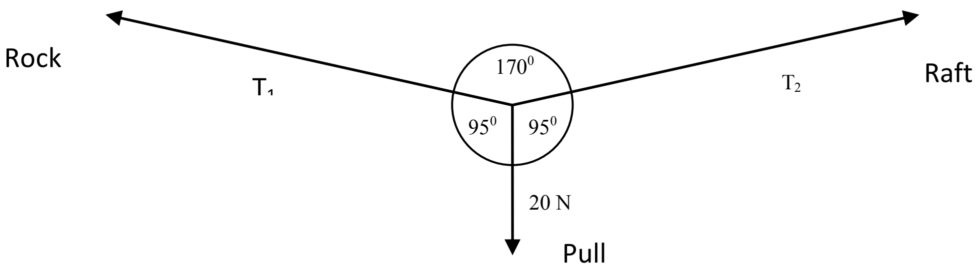


ΣF = $\sqrt{}(36.38^2 + 715.27^2)$
ΣF **= 716 N**

Tan θ = 715.27 / 36.38
θ = 87.1^0

ΣF **= 716 N at 87.1^0 to the back of the boat or 2.91^0 below the mid line.**

23.



Using sin rule
$20 / \sin 170^o = T_2 / \sin 95^o$

T_2 = 115 N **The closer the 170^0 angle is to 180^0, the larger the Tension in the wire.**

Problem Set 2: Moments and equilibrium

1. m N is N m which is newton meters. These are the units for torque (moment)
   mN is milli newtons. This is newtons x 10^{-3}.

2. M = F x r
   M = 160 x 0.75
   M = 120 N m
   (direction cannot be specified other than having a knowledge of nuts and bolts. To tighten bolts you must turn them clockwise).

3. M = F x r
   88= F x 0.4
   F = 220 N at right angles to the wrench.

4.

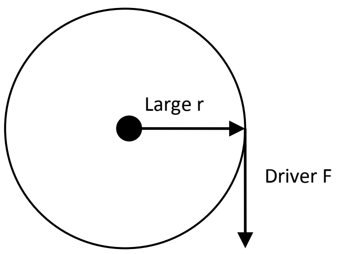
Truck

The truck has a very ―heavy” steering mechanism. As a consequence it will require a lot of torque to get it to turn.

By increasing the diameter of the steering wheel, the torque that can be created when the driver applies their force to the wheel will be maximised.

The speed with which the driver is required to turn the wheel is not a major consideration

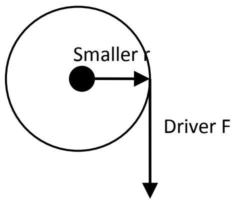
A large wheel provides a force (torque advantage).



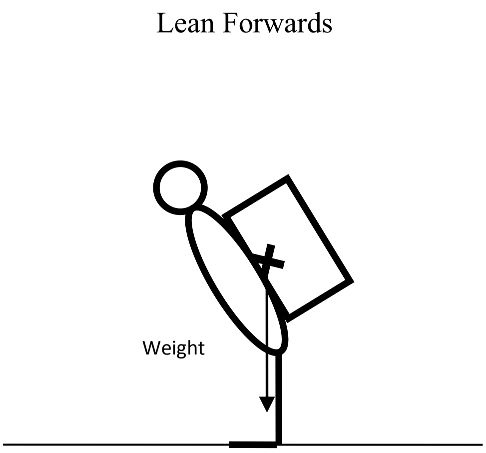
Car

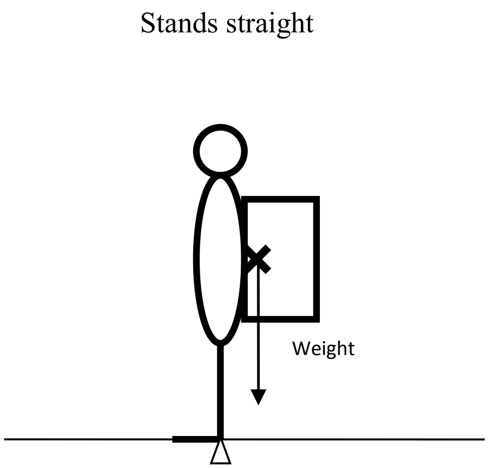
The racing car has a very light steering mechanism. What is important is that the driver be able to respond quickly to changes in conditions in front of them. A small steering wheel does not have to be turned through a large distance in order to bring about a change in the direction of the vehicle.

A small radius steering wheel provides a shortness of turning distance advantage.



5. Michael leans forward to keep the combined centre of mass (his com and the backpacks com combined) above his base. This causes the weight vector of the combined com to act through the base eliminating any toppling torque. If he does not lean forward the weight of the combined centre of mass acts outside his base (feet behind the heels) and this causes him to topple over backwards.

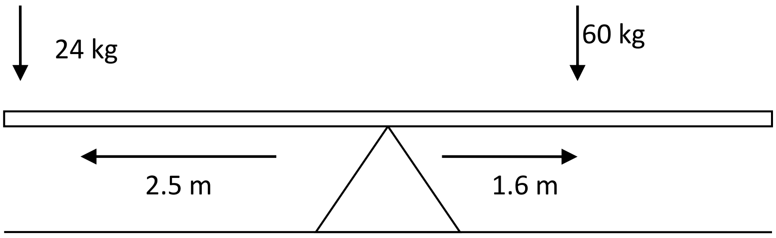




Does not topple – weight within base

Does topple – weight outside of base

6.a)

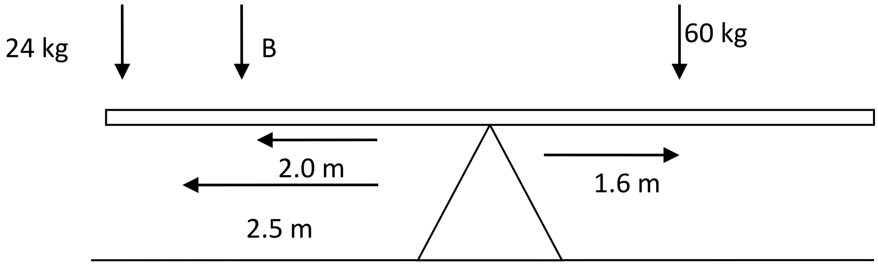


$\Sigma M_c = 60 \times 9.8 \times 1.6$
$\Sigma M_c = 940.8$ N m Clockwise

$\Sigma M_a = 24 \times 9.8 \times 2.5$
$\Sigma M_a = 588.0$ N m Anti clockwise

940.8 N m Clockwise ≠ 588.0 N m Anti clockwise and so cannot reach a balance.

6.b)



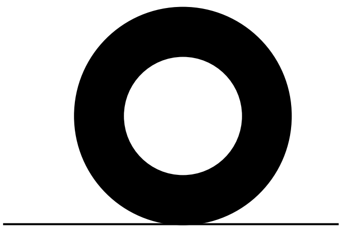
$\Sigma M_c = \Sigma M_a$

(60 x 9.8 x 1.6) = (24 x 9.8 x 2.5) + (B x 9.8 x 2.0)
940.8 = 588.0 + 19.6 B
940.8 - 588.0 = + 19.6 B
940.8 - 588.0 = + 19.6 B
352.8 / 19.6 = B

B = 18.0 kg

7. In both situations the torque provided by the motor is the same.

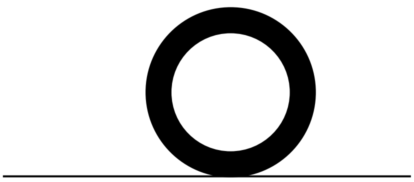
Standard Tyres (large radius)



M (constant) = r↑ F↓.

For a constant torque the larger the radius the smaller the force. The smaller the force the smaller the acceleration of the car by F = ma

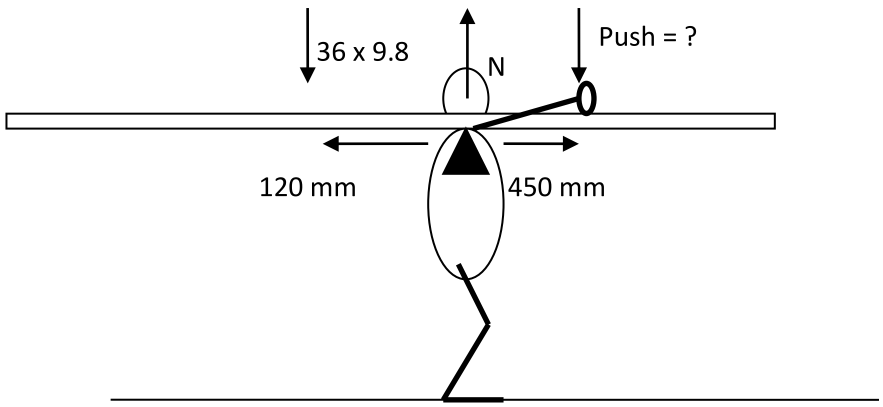
Low Profile Tyres (smaller radius)



M (constant) = r↓ F↑.

For a constant torque the smaller the radius the larger the force. The larger the force the greater the acceleration of the car by F = ma

8.



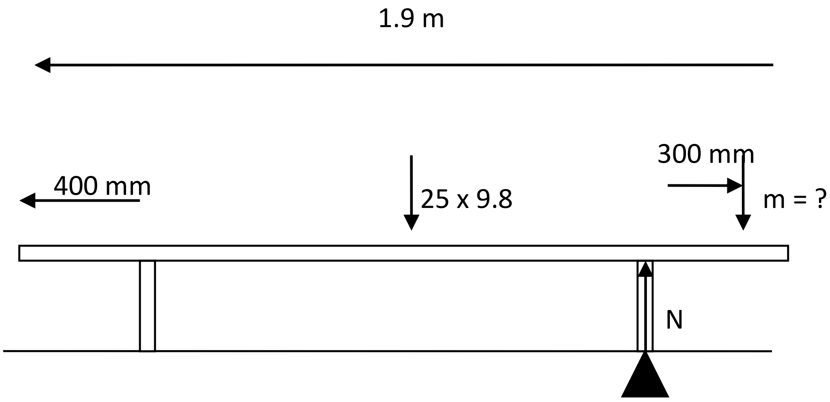
a)
Σ M c = Σ M a
Σ 0.120 x 36 x 9.8 = Push x 0.450

Push = 94.1 N down

b)
Σ F up = Σ F down
N = 36 x 9.8 + 94.1 N

N = 447 N up

9.

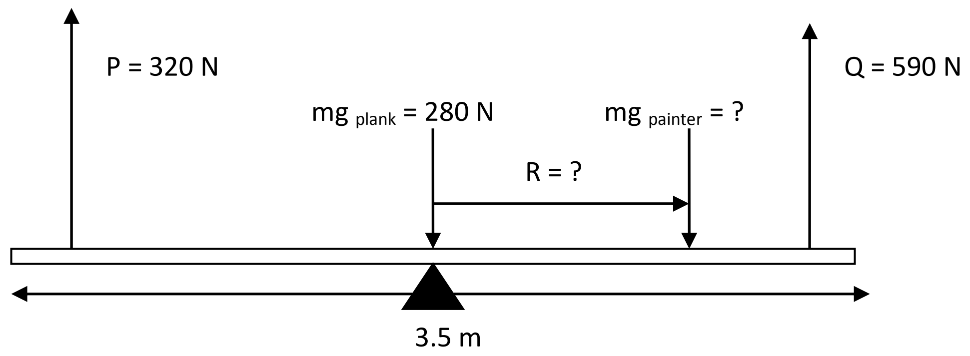


The pivot must be placed at the leg because the seat is in unstable equilibrium

Σ M a = Σ M c
0.55 x 25 x 9.8 = m x 9.8x 0.300

m = 46 kg

10.



a)

Σ F up = Σ F down   320 + 590 = 280 + mg $_{painter}$ **mg $_{painter}$ = 630 N Down**

b) Take moments about the centre of the platform. Assume that the ropes are attached to the ends of the plank.

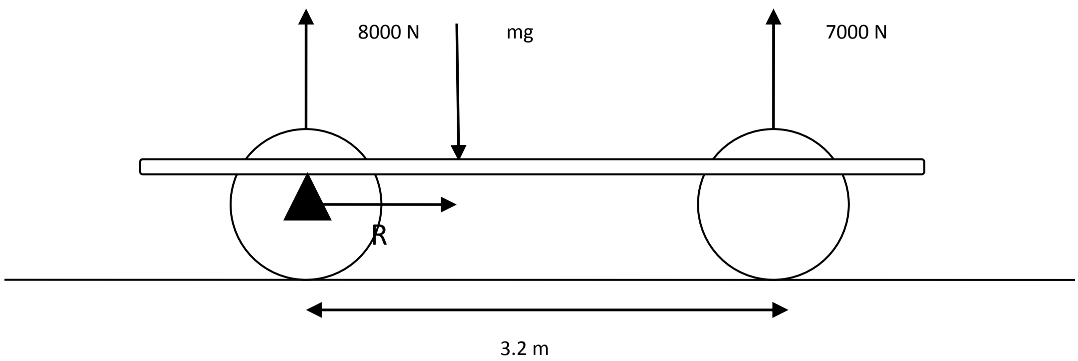
Σ M c = Σ M a   [320 x (3.5/2)] + [630 x R] = [590 x (3.5/2)]

560 + 630R = 1032.5

$R = \frac{1032.5 - 560}{630}$

R = 0.75 m towards Q

11.



Σ F up = Σ F down

8000 + 7000 = mg

mg = 15000 N Down

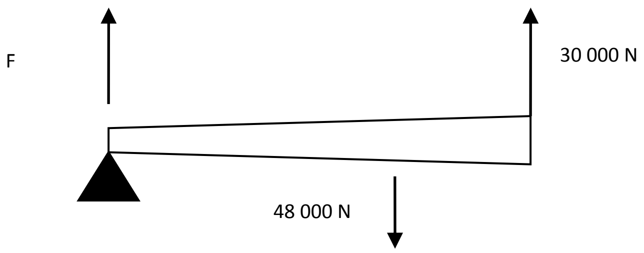
Take moments about front wheel of car.

Σ M c = Σ M a

[15000 x (R)] = [7000 x 3.2]

R = 1.49 m from the front wheel

12. a) Σ F up = Σ F down
30 000 + F = 48 000
F = 18 000 N

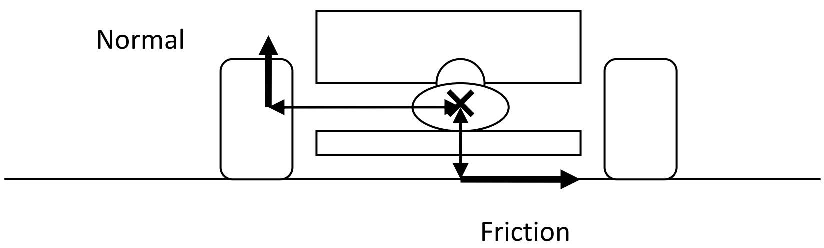


b) Let the length of the log be 1.00 m. Take moments about the light end
Σ M c = Σ M a [48 000 x (R)] = [30 000 x 1]
R = 0.625 m from the light end (R = 0.375 m from the heavy end)
(62.5 % from the light end or 37.5 % from the heavy end if length is uspecified.)

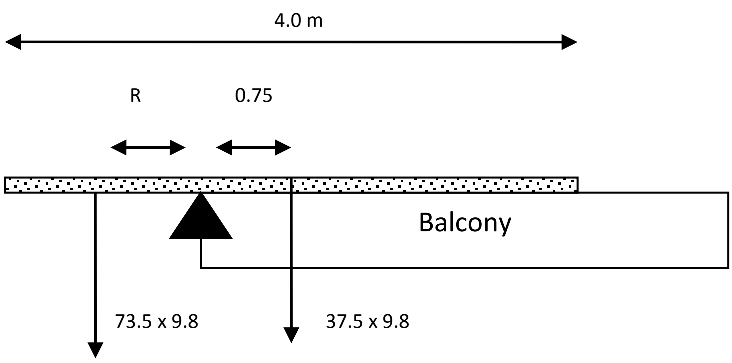
13. When a car goes around a corner on a flat road it is the outside tyres that tend to provide the centripetal force by friction required to round the bend. If the torque provided by the outer tyres accelerating towards the centre of the curve is greater than the torque provided by the normal force acting on the tyre, the racing car will roll over.

Because the racing car is accelerating around the bend, torques must be taken about the centre of mass of the object. This is different to when the object is

- in stable equilibrium – the pivot can be chosen arbitrarily.
- in unstable equilibrium – the pivot is taken about the point base.



14. a)
Torques must be taken about the edge of the balcony because just as the plank is about to tip the plank is in unstable equilibrium.



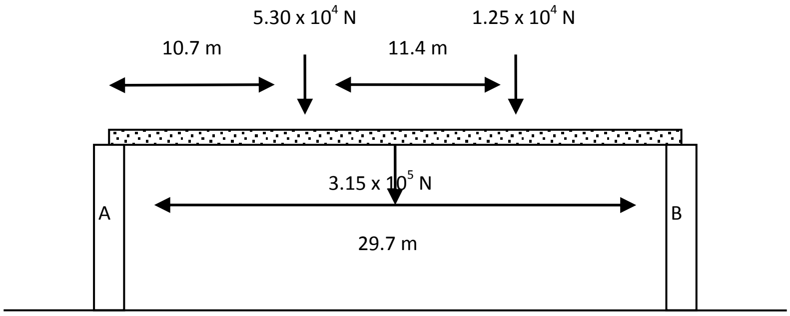
Σ M c = Σ M a

[37.5 x 9.8 x 0.75] = [73.5 x 9.8 x R]
[275.625] = [720.3 x R]
$\frac{275.625}{720.3}$ = x R

R = 0.383 m from the edge of the balcony

b) Move the com of the plank further form the edge or put the paint can at the opposite end of the plank to add extra stabilising torque to the plank.

15.



Take moments about pier A.
Σ M c = Σ M a
$(5.3 \times 10^4 \times 10.7) + (1.25 \times 10^4 \times (10.7 + 11.4)) + (3.15 \times 10^5 \times (29.7/2)) = [B \times 29.7]$
$(5.671 \times 10^5 + 2.7625 \times 10^5 + 4.67775 \times 10^6) = 29.7\ B$
$B = 1.86 \times 10^5$ N up

Σ F up = Σ F down
$A +$ **$1.86 \times 10^5 =$** $5.3 \times 10^4 + 1.25 \times 10^4 + 3.15 \times 10^5$
$A = 1.95 \times 10^5$ N up

16. With the backs of their legs to the wall, as they bend forward, the centre of mass of their body is put outside their base and they topple forward. They can only achieve this if they can keep the com inside their base (feet) at all times which is impossible … so they fall / topple.

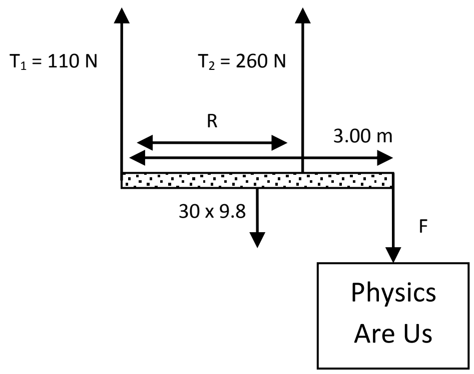
    You should draw a diagram to support your answer.

17. While your com is closer to the ground, the size of you base (hands) is much smaller than usual (your feet). This results in you (the handstand) being less stable than usual. You could do a toppling angle analysis on the basis of (base / height to com) = tan θ

18. You need to shift your centre of mass from side to side to keep it above the foot that is on the ground to avoid inducing a torque that causes you to topple.

19. By leaning forward as you pass over the hurdle, it minimises the fluctuation in the change in height of the centre of mass of the hurdler. When the com of the hurdler rises, the potential energy of the hurdler increases. By the law of conservation of energy, if your potential energy increases then your kinetic energy and consequently velocity decreases. A slow velocity causes you to travel the distance of the race in a longer time. Hence you have a greater chance of losing.

20.

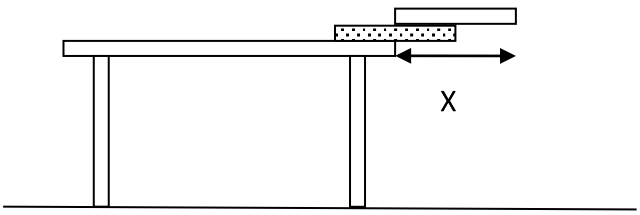


Σ F up = Σ F down
110 + 260 = 294 + F
F = 76 N Down

m =F / g
m = 76 / 9.8
m = 7.76 kg

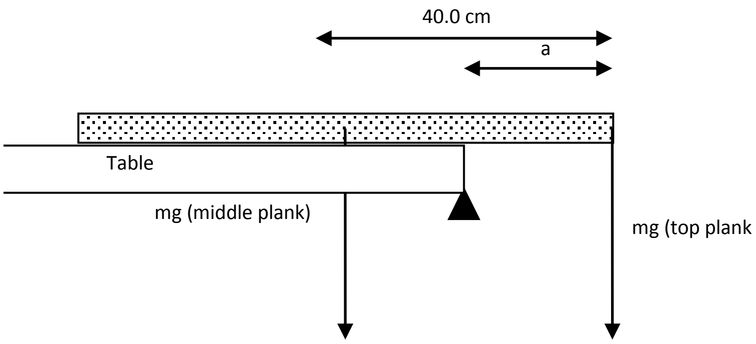
Σ M c = Σ M a
(294 x 1.5) + (76 x 3) = [260 x R]
441 + 228 = 260 R
R = 669 / 260
R = 2.57 m from T_1 end

21. Hint - analyse the plank that is in between the table and the top plank because it touches all other objects.



Place the COM of the top plank at the edge of the middle plank.
The pivot is at the edge of the table because the system is in unstable equilibrium.

The free body diagram of the middle plank (close up view) is thus …



Σ M c = Σ M a
[mg x a] = [mg x (0.4 – a)]
mga = 0.4mg - mga
2mga = 0.4mg
2a = 0.4
a = 0.4 / 2
a = 0.2 m

The plank on top of the middle plank sticks out an additional 0.4 m.

Hence the distance X is 0.6 m (0.4 + 0.2)

22. The critical factor is that the centre of mass of the wood bottle system is above the base so that the weight force acts through the base. If the weight force did not act through the base, a torque would be induced that would topple the system.

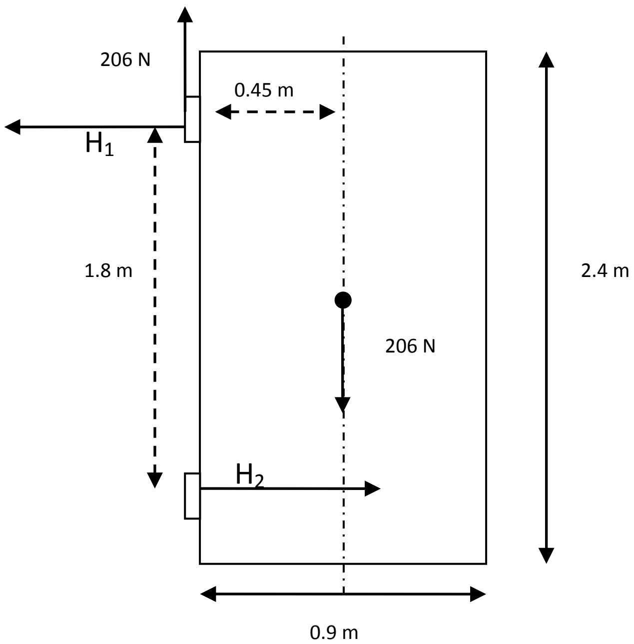
23. a)

Take moments about top hinge

$\Sigma M_c = \Sigma M_a$

$0.45 \times 206 = 1.8 \times H_2$

H_2 = 51.5 N to the right (diagram)



23. b)

$\Sigma F_{left} = \Sigma F_{right}$

$H_1 = H_2$

H_1 = 51.5 N Left

$\Sigma F_{up} = \Sigma F_{Down}$

103 + 103 = 206

H_1 Resultant

Magnitude

H1 Resultant^2 = $(103^2 + 51.5^2)$

H_1 Resultant = 115 N

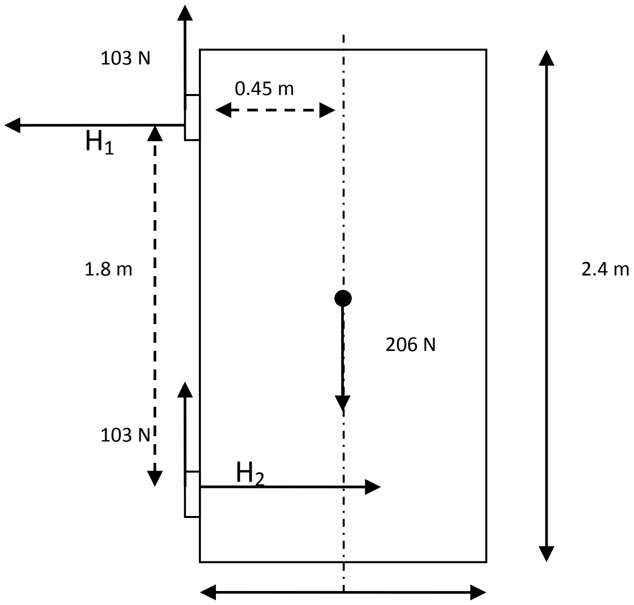
Angle

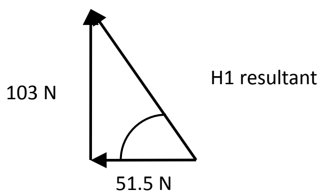
Tan θ = 103 / 51.5

θ = 63.4 ^{0}

Ans H_1 Resultant = 115 N left 63.4 ^{0} up

H_2 by symmetry is 115 N right 63.4 ^{0} up.





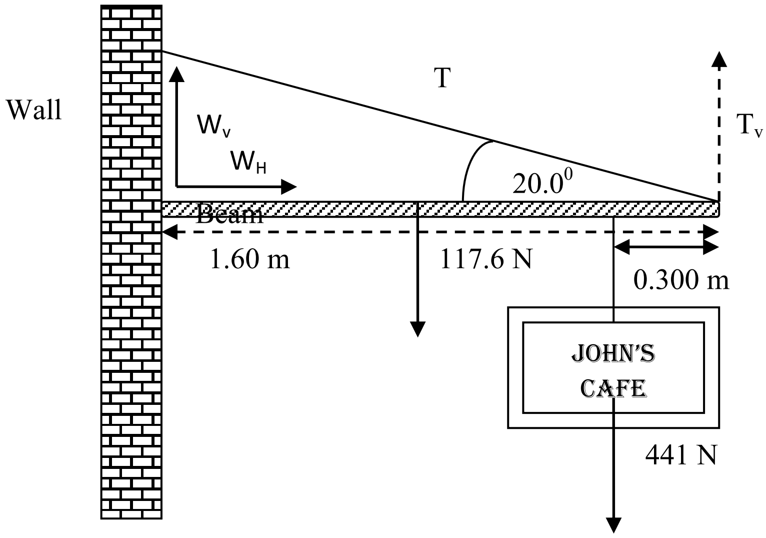
24. **a)** Take moments about hinge at wall beam intersection

$\Sigma M_c = \Sigma M_a$

$(0.8 \times 117.6) + (1.3 \times 441) = (1.6 \times T_v)$

$(94 + 573.3) / 1.6 = T_v$

$\mathbf{T_v = 417\ N}$



$\text{Cos } 70 = T_v / T$

$T = 417 / \text{Cos } 70$

$\mathbf{T = 1219\ N}$

$\mathbf{T = 1.22 \times 10^3\ N}$

along the wire.

b) $\Sigma F_{up} = \Sigma F_{down}$

$W_v + 417 = 117.6 + 441$

$\mathbf{W_v = 141.6\ N}$

$\Sigma F_{left} = \Sigma F_{right}$

$T_h = W_h$

$T \text{ Cos } 20 = W_h$

$1219 \text{ Cos } 20 = W_h$

$\mathbf{W_h = 1145\ N}$

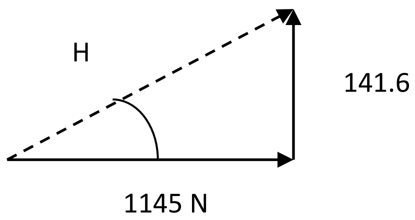
c)

$W = (1145^2 + 141.6^2)$

$W = 1153.7\ N$

$\text{Tan } \sigma = 141.6 / 1145$

$\sigma = 7.05^0$



W = 1.15 x 10^3 N Right 7.05^0 Up

25. **Find Tension**

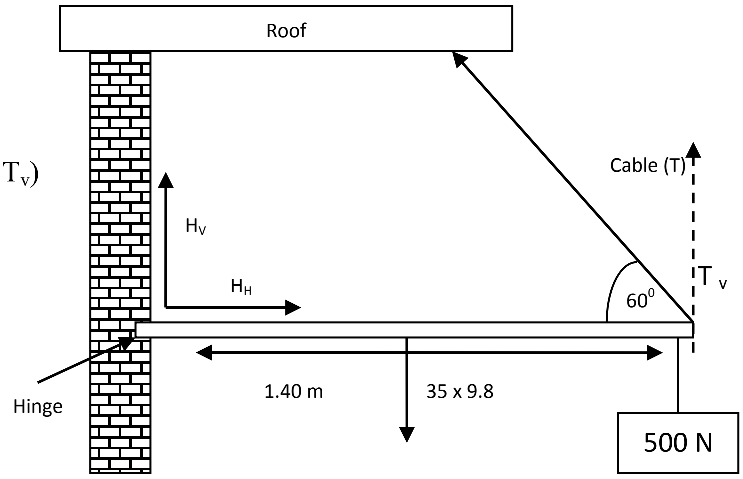
Take moments about hinge

$\Sigma Mc = \Sigma Ma$

$(0.7 \times 35 \times 9.8) + (500 \times 1.4) = (1.4 \times T_v)$

$(240.1 + 700) / 1.4 = T_v$

T_v **= 671.5 N**



$\text{Cos } 30 = T_v / T$

$\text{Cos } 30 = 671.5 / T$

$T = 671.5 / \text{Cos } 30$

T = 775.4 N along the wire

Find the force of the hinge on the beam

Σ F up = Σ F down

$H_v + T_v = (35 \times 9.8) + (500)$

$H_v + 671.5 = (343) + (500)$

H_v = 171.5 N up

Σ F left = Σ F right

$H_h = T_h$

Cos 60 = T_h / T

T_h = T x Cos 60

T_h = 775.4 Cos 60

T_h = 387.7 N Left

$H_h = T_h$

H_h = 387.7 N Right

Combine the H_v and H_h

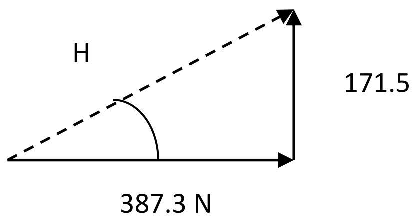
Pythagoras

$H = (171.5^2 + 387.7^2)^{½}$

H = 424 N

Tan θ = 171.5 / 387.3

$θ = 23.9^0$



Answer = W = 424 N Right 23.9^0 Up

26. Σ Mc = Σ Ma

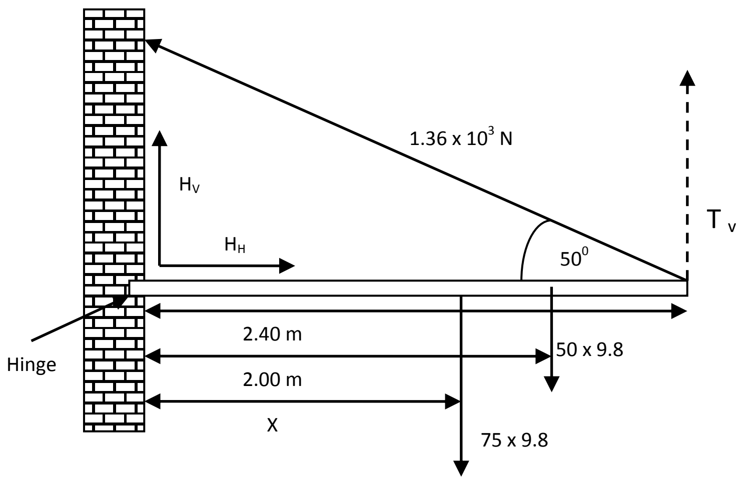
$(2.00 \times 50 \times 9.8) + (X \times 75 \times 9.8) = (2.4 \times 1.36 \times 10^3 \times \cos 40^0)$

980 + 735X = 2500

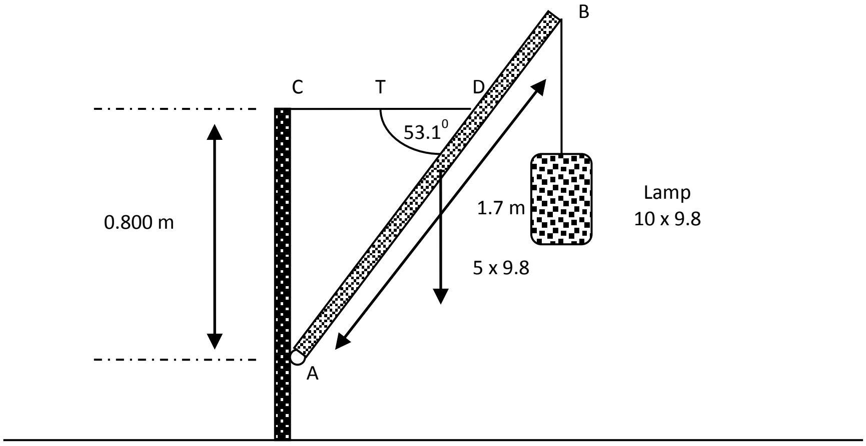
735X = 2500 - 980

X = (2500 – 980) / 735

X = 2.07 m



27.



a) Find tension

Σ Mc = Σ Ma

$([1.7 / 2] \cos 53.1^0 \times 5 \times 9.8) + (1.7 \cos 53.1^0 \times 10 \times 9.8) = (T \times 0.8)$

25 + 100 = 0.8 T

125 / 0.8 = T

T = 156 N

b) Forces at A

Σ F up = Σ F down

$A_v = (5 \times 9.8) + (10 \times 9.8)$

A_v = 147 N up

Σ F left = Σ F right

$T = A_h$

A_h = 156 N Right

Combine the A_h and A_v

Pythagoras

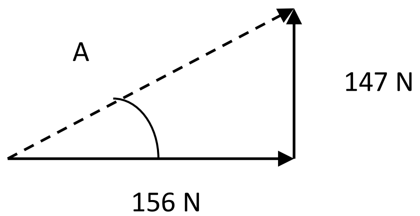
$A = (156^2 + 147^2)^{½}$

A = 214 N

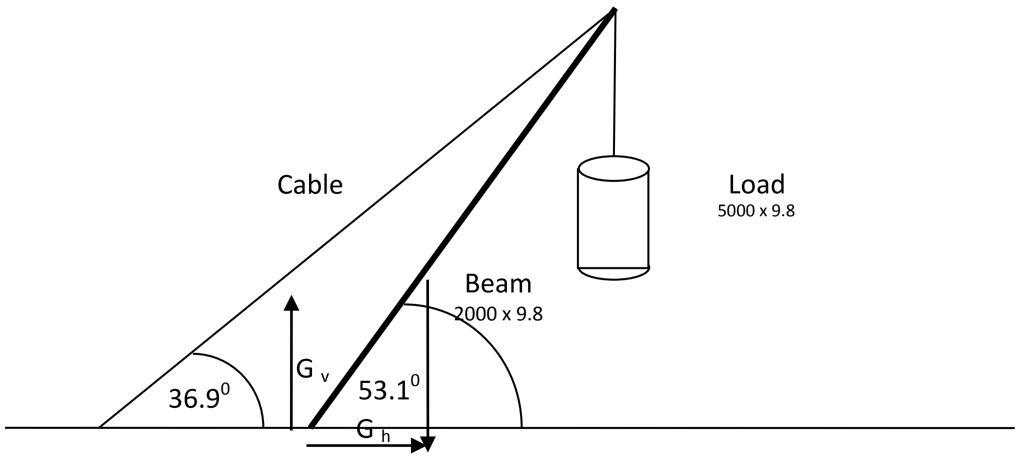
$\tan \theta = 147 / 156$

$\theta = 43.3^0$

Answer = A = 214 N Right 43.3^0 Up



28.



a) No dimensions are given.

Let the beam = 3 m.
(This is convenient because the COM of the beam is 1/3 rd of the way up from the ground).

Take pivots about the base of the beam, where the forces are not known

Σ Mc = Σ Ma
([1 Cos 53.1^0 x 2000 x 9.8) + (3 Cos 53.1^0 x 5000 x 9.8) = (3 x T_{perp})
$1.176 \times 10^4 + 8.826 \times 10^4 = 3 \times T_{perp}$
$3.334 \times 10^4 = T_{perp}$

Angle in the top corner is 16.2^0
Cos (73.8^0) = T_{perp} / T
Cos (73.8^0) = 3.334×10^4 / T
T = 1.195×10^5 N

b) Forces at G
Σ F up = Σ F down
G_v = (2000 x 9.8) + (5000 x 9.8) + T_v

Cos 53.1 = T_v / 1.195×10^5
T_v = 1.195×10^5 N x Cos 53.1
$T_v = 7.175 \times 10^4$ N

G_v = (2000 x 9.8)+(5000 x 9.8)+7.175×10^4
$G_v = 1.404 \times 10^5$ N

Σ F left = Σ F right
$G_h = T_h$

Cos 36.9 = T_h / 1.195×10^5

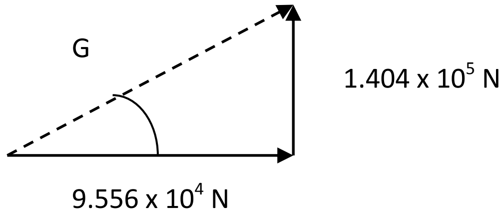
T_h = 1.195×10^5 x Cos 36.9
$T_h = 9.556 \times 10^4$ N

$G_h = 9.556 \times 10^4$ N

Combine the G_h and G_v

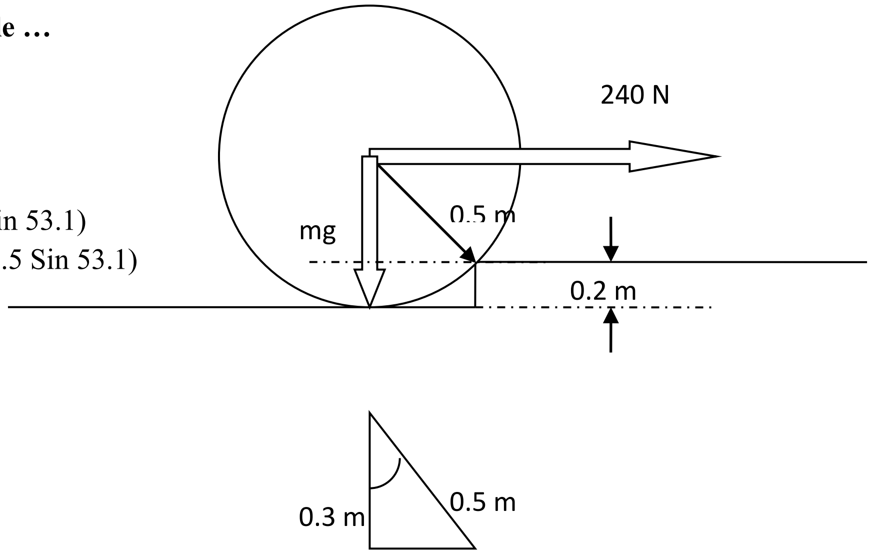
Pythagoras
$G = (9.556 \times 10^{4\,2} + 1.404 \times 10^{5\,2})^{½}$
$G = 1.698 \times 10^5$ N

Tan θ = $1.404 \times 10^5 / 9.556 \times 10^4$
θ = 55.76^0
Answer = G = 1.70×10^5 N Right 55.8^0 Up



29. **a)** **From the triangle …**
Cos θ = 0.3 / 0.5
θ = 53.1

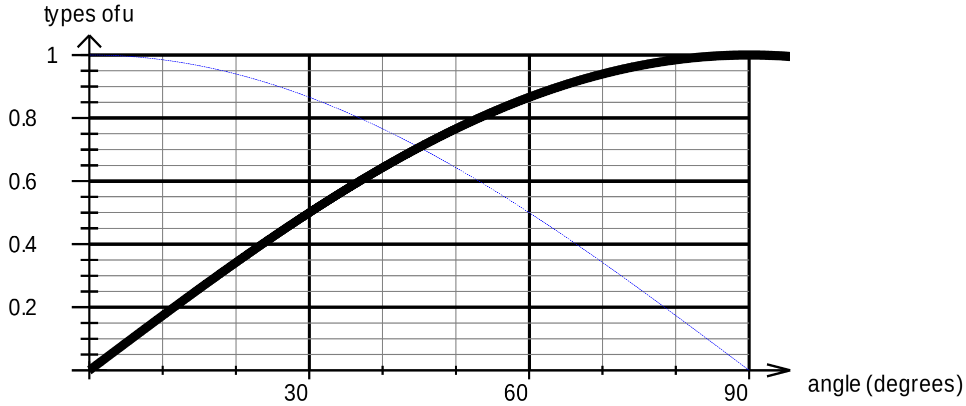
Σ M clock= Σ M anti
(240 x 0.3) = (mg x 0.5 Sin 53.1)
(240 x 0.3) = (m x 9.8 x 0.5 Sin 53.1)
72 = m x 3.91845
72 / 3.91845 = m
m = 18.4 kg



b)
The force applied will be minimised if it is applied at right angles to the distance
Torque applied by 240 N is …
(240 x 0.3) = 72 N m
The alternative is to apply a smaller force at right angles at a distance of 0.5 m
72 = 0.5 x F at right angles
F = 144 N at 53.1 degrees above horizontal.

Problem Set 3: Projectile motion and air resistance

1. The launch angle of 45.0^0 is a compromise between maximising the vertical and horizontal components of the launch velocity. Increasing the vertical component increases the flight time before landing. Increasing the horizontal component increases the horizontal distance covered while the projectile is in the air. A launch angle of 45.0º - which only maximises the range when the take-off and landing heights are the same – optimises the interaction between these two quantities. For projectiles in which the landing height is lower than take-off, the range is maximized by using an angle slightly lower than 45.0^0. For landing heights higher than take-off height the range is maximized by using an angle slightly above 45.0^0.

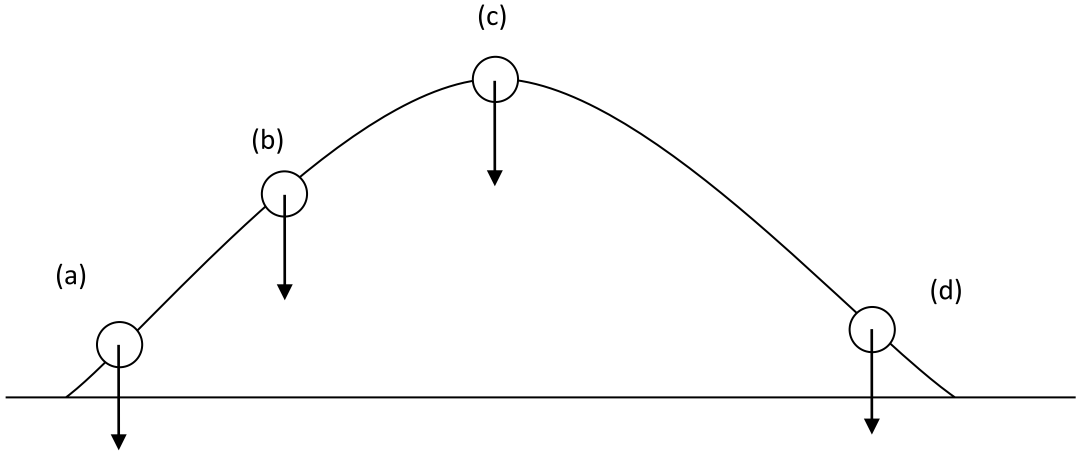


solid line = $u_v = u \sin$ (angle)
dotted line = $u_h = u \cos$ (angle)

45.0^0 is the point where the two functions are maximised. (linear programming)

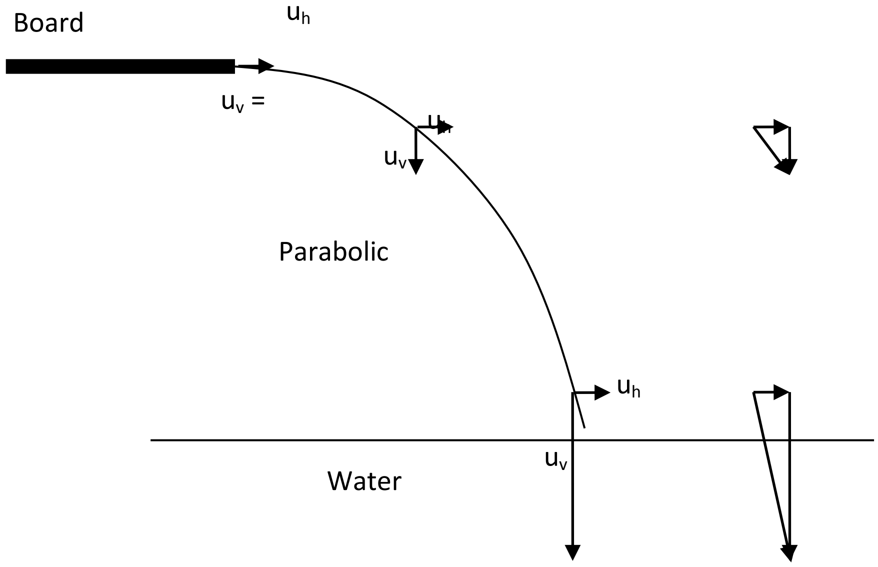
2. If the force of air resistance is ignored, then there is only one force acting on the ball – gravity. The force of gravity (weight) acts vertically downwards towards the ground – it has no horizontal component. Therefore, there is no component of the weight force that can accelerate the ball in a horizontal direction. Hence, if air resistance is ignored, the velocity of the ball in the horizontal plane will be constant.

3.



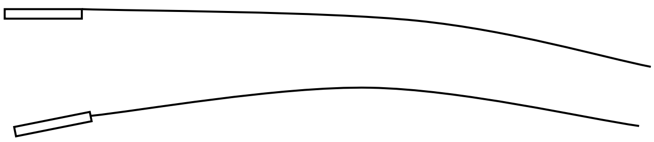
All the forces (weight of the ball) are the same size (ie, each vector should be the same length and act vertically downwards). No other force vectors should be drawn.

4.

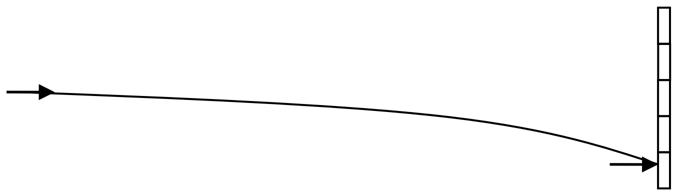


Note: u_h vectors should be the same length and horizontal in direction throughout the diver's flight.

5. Throughout its flight, the bullet is accelerated towards the ground by the force of the bullet's weight. This causes the bullet to lose height. By aiming above the target, the bullet will follow a parabolic trajectory; the bullet will rise to a maximum height and then fall back to the height at which it will hit the target.



6. This question requires that you assume the original height of the arrow from the ground and that the centre of the target is at the same height.

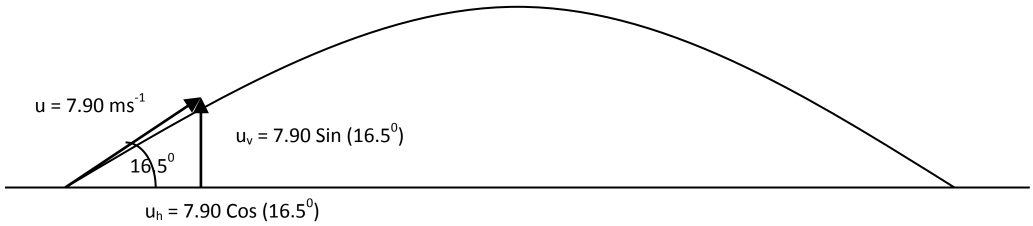


Vertical	Horizontal
$a = -9.8\ ms^{-2}$	$u_h = 80\ ms^{-1}$
$s = ?$	$s_h = 16\ m$
$s = ut + \frac{1}{2} at^2$	$u = s / t$
$s = 0 + \frac{1}{2} \times -9.8 \times 0.200^2$	$t = 16 / 80$
$s = -0.196\ m$	**$t = 0.200\ s$**

ORANGE: r_1 = 0.0600 m; YELLOW: r_2 = 0.0600 m – 0.180 m; BROWN: r_3 = 0.180 m – 0.420 m

∴ Arrow lands in the BROWN zone = 0.016 m (scores 5 points)

7. In this question, assume that the long jumper takes off and lands at the same height



Vertical	Horizontal
$a = -9.8\ ms^{-2}$	$u_h = 7.90\ Cos\ (16.5^0) = 7.57\ ms^{-1}$
$u_v = 7.90\ Sin\ (16.5^0) = 2.24\ ms^{-1}$	$t = 0.457\ s$
	$s_h = ?$
$u = 2.24\ ms^{-1}$; $v = -2.24\ ms^{-1}$; $a = 9.80\ ms^{-2}$	
	$7.57 = s / 0.457$
$v = u + at$	$s = 7.57 \times 0.457$
$-2.24 = +2.24 + (-9.80)\ t$	
$-4.48 / -9.80 = t$	**$s = 3.46\ m$**
$t = 0.457\ s$	

8.

	Throw as fast as possible	Throw slower
Advantages	The force with which the ball hits the tins will be larger and directed horizontally through the cans with only a small component in the vertical.	The air resistance experienced by the balls will be smaller and will be less likely to steer the ball off course, causing it to miss the target.
Disadvantages	The air resistance on the ball will be larger due to the increased velocity. This air resistance, if not evenly distributed, will cause the ball to steer off course and slow dramatically.	The force with which the ball hits the cans will be reduced; and the horizontal component of that force will also be smaller because the ball will be on a steeper trajectory downwards when it hits the cans.

9. $u_v = 4.00\ ms^{-1}$
$s = -1.10\ m$
$a = -9.80\ ms^{-2}$
$v = ?$
$t = ?$

$v^2 = u^2 + 2as$
$v^2 = 4.00^2 + 2 \times -9.80 \times -1.10$
$v^2 = 16 + 21.6$
$v^2 = 37.6$
$v = -\ 6.13\ ms^{-1}$

$v = u + at$
$-\ 6.13 = 4.00 + (-9.80)t$
t = 1.03 s

10. The ball takes off and lands at a different height.

Vertical

$a = -\ 9.80\ ms^{-2}$
$s = ?$
$u = 22.5 \text{ Sin } 10^o =$ **3.91 ms^{-1}**

$s = ut + ½ at^2$
$s = (3.91 \times 0.874) + ½ \times -9.80 \times 0.874^2$

s = -0.326 m below the release height

1.50 + (– 0.326) **= 1.17 m above the ground.**

Horizontal

$u = 22.5 \text{ Cos } 10^o =$ **22.2 ms^{-1}**
$s_h = 19.4\ m$

$u = s / t$
$t = 19.4 / 22.2$
t = 0.874 s

11. a)

Vertical
$a = -\ 9.8\ ms^{-2}$
$u_v = 18 \text{ Sin } (35^0) = 10.3\ ms^{-1}$
$v_v = -10.3\ ms^{-1}$ ($s_v = 0$ m)

$v = u + at$
$-10.3 = +\ 10.3 + (-9.80)\ t$
t = -20.6 / -9.80

t = 2.10 s

b)

Horizontal
$u_h = 18 \text{ Cos } (35^0) = 14.7\ ms^{-1}$
$t = 2.10\ s$
$s_h = ?$

$v = s / t$
$14.7 = s / 2.10$
$s = 14.7 \times 2.10$

s = 30.9 m for the start

12. Assume take-off and landing heights are the same.

(25º above horizontal)

Vertical
$a = -\ 9.80\ ms^{-2}$
$u_v = 28 \text{ Sin } (25^0) = 11.8\ ms^{-1}$
$v_v = -11.8\ ms^{-1}$
$t = ?$

$v = u + at$
$-11.8 = +\ 11.8 + (-9.80)\ t$
-23.6 / -9.8 = t
t = 2.41 s

Horizontal
$u_h = 28 \text{ Cos } (25^0) = 25.4\ ms^{-1}$
$t = 2.41\ s$
$s_h = ?$

$v = s / t$
$s = v\ t$
$s = 25.4 \times 2.41$

s_h = 61.2 m

(40º above horizontal)

Vertical

a = - 9.80 ms^{-2}
u_v = 28 Sin (40^0) = 18.0 ms^{-1}
v_v = -18.0 ms^{-1}

v = u + at
-18.0 = 18.0 + (-9.80) t
-36.0 / -9.80 = t

t = 3.67 s

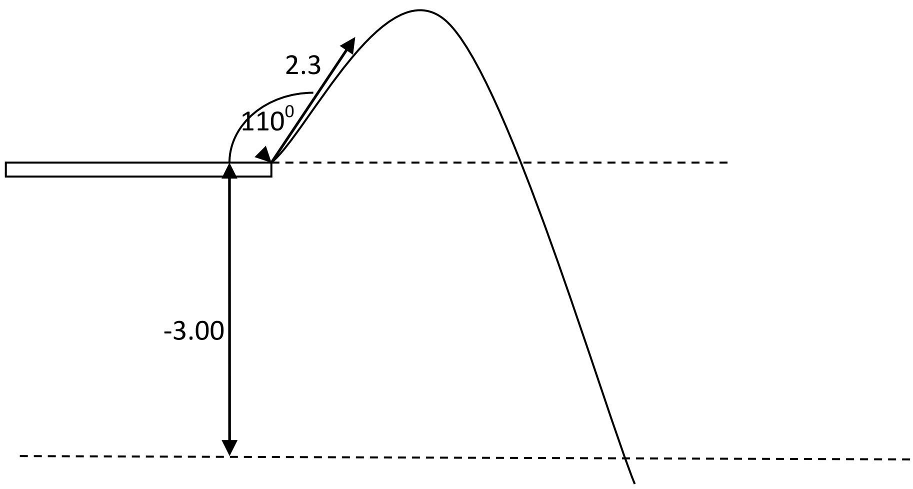
Horizontal

u_h = 28 Cos (40^0) = 21.4 ms^{-1}
t = 3.67 s
s_h = ?

v = s / t
s = v t
s = 21.4 x 3.67

s_h = 78.5 m
(this will be the longest throw she can achieve)

13.



Vertical

u_v = 2.30 Sin 70º = 2.16 ms^{-1}
s_v = -3.00 m
a = -9.80 ms^{-2}
t = ?

$v^2 = u^2 + 2as$
$v^2 = 2.16^2$ + 2 x -9.80 x -3.00
v^2 = 4.67 + 58.8
v^2 = 63.5
v = - 7.97 ms^{-1}

v = u + at
- 7.97 = (2.16) + (-9.80)t
t = 1.03 s

Horizontal

u_h = 2.30 cos 70º = 0.787 ms^{-1}
t = 1.03 s
u_h = s / t

0.787 = s / 1.03

s_h = 0.811 m

14. **Vertical**

$u_v = 0\ ms^{-1}$
$s_v = ?$
$a = -9.80\ ms^{-2}$
$t = 7.62\ s$

$s_v = ut + ½\ at^2$
$s_v = 0 + ½\ (-9.80)\ (7.62)^2$
$s_v = -285\ m$

Hence, water must be released 285 m above the ground.

Horizontal

$u_h = 21.0$ ms-1
$s_h = 160\ m$

$u_h = s / t$
$t = s / u_h$
$t = 160 / 21.0$

$t = 7.62\ s$

15. a) **Vertical**

$u_v = 11.0 \sin 55^0 =$ **$9.01\ ms^{-1}$**

Horizontal

$U_h = (11.0 \cos 55^0) + 2.80$
$= 6.31 + 2.80$
$=$ **$9.11\ ms^{-1}$**

b) **Vertical**

$u_v =$ **$9.01\ ms^{-1}$**
$v_v = 0$
$a = -9.80\ ms^{-2}$
$s_v = ?$

$v^2 = u^2 + 2as$
$0^2 = 9.01^2 + 2\ (-9.80)\ s$
$s_v = 4.14\ m$

$\therefore$ Max h $_{above\ ground} = 4.14 + 2.40$
$= 6.54\ m$

Horizontal

b) **Vertical**

$u_v = 9.01\ ms^{-1}$
$a = -9.80\ ms^{-2}$
$v_v = ?$
$s_v = -2.40\ m$

$v^2 = u^2 + 2as$
$v^2 = 9.01^2 + 2(-9.80)(-2.40)$
$v_v = -11.3\ m$

$v = u + at$
$-11.3 = 9.01 + (-9.80)\ t$
$t = 2.07\ s$

Horizontal

$u_h = 9.11\ ms^{-1}$
$t = 2.07\ s$

$u_h = s / t$
$s_h = u_h\ t$
$s_h = 9.11 \times 2.07$

$s = 18.9\ m$

16. a) 108 / 3.6 = 30.0 ms^{-1}

Vertical	**Horizontal**
u_v = 30.0 Sin 15° = **7.76 ms^{-1}**	u_h = 30.0 Cos 15° = **29.0 ms^{-1}**

b) No. During the flight, the car will always be experiencing an acceleration of 9.80 ms^{-2} vertically downwards due to gravity.

c) At its highest point (maximum height) the velocity will be 29.0 ms^{-1} in a horizontal direction. At this point, the vertical component of its velocity is zero. Hence, its velocity will only consist of its original horizontal launch component (if friction is ignored).

d) Given that the jump is completed successfully with a **minimum** speed, we can assume that the car lands at D. There are a few possibilities here:

If air resistance is ignored, then the car will land at D with the same speed as at A and B. If the driver does not apply any brakes when it lands, the car will accelerate down the slope and will experience its greatest speed at E.

If air resistance is NOT ignored, then the car will land at D with a lower speed than at A and B. Again, if the driver does not apply any brakes when it lands, the car will accelerate down the slope and will experience its greatest speed at E.

In both scenarios, minimum velocity will be at C (v_v = 0 ms^{-1}).

There may be other scenarios.

e) Assume that B and D are at the same height; let launch speed be _v'.

Vertical

u_v = v Sin 15°
t = 39.3 / v
a = -9.80 ms-2
s_v = 0 m

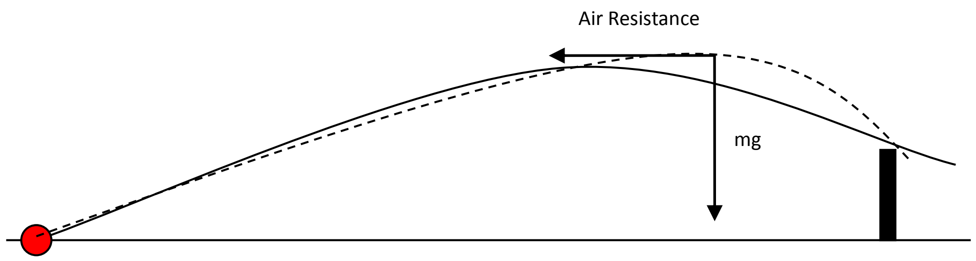
s = ut + ½ at^2
0 = (v Sin 15) (39.3 / v) + 0.5(-9.80)$(39.3 / v)^2$
0 = 10.2 – 7568 / v^2

v^2 = 742
v = **27.2 ms^{-1}**

Horizontal

u_h = v cos 15°
s_h = 38.0 m
u_h = s_h / t
u_h = 38.0 / v cos 15°
t = 39.3 / v

17. a)



b) Solid line is without air resistance.
Dotted line is with air resistance

c) **Vertical** | **Horizontal**

$u_v = 28.0 \sin 35^o = 16.06 \text{ ms}^{-1}$
$a = -9.80 \text{ ms}^{-2}$
$t = 2.79$ s
$s_v = ?$

$s = ut + ½ at^2$
$s = (16.06 \times 2.79) + ½ (-9.80) (2.79)^2$
s = **6.67 m;** so it clears the 1.4 m high fence by a height of 5.27 m.

$u_h = 28.0 \cos 35^o = 22.94 \text{ ms}^{-1}$
$s_h = 64.0$ m
$u_h = s_h / t$

$t = 64.0 / 22.94$
t = 2.79 s

The alternative way of working this question is finding out when the ball will be at a height of 1.4 m and then working backwards to determine if the range is greater than 64 m. This is the harder way of working the question. (2 possible solutions)

18. **Vertical** | **Horizontal**

$u_v = 0 \text{ ms}^{-1}$
$s_v = -(1.35 – 0.45) = - 0.90$ m

$s = ut + ½ at^2$
$-0.90 = (0) + ½ (-9.80) (t)^2$
t = 0.429 s

$u_h = 83.0 \text{ ms}^{-1}$
$t = 0.429$ s
$s_h = ?$

$u_h = s_h / t$
$s_h = u_h \times t$
$\therefore s = 83.0 \times 0.429$
$\mathbf{s_h = 35.6 \text{ m}}$

19. Let launch speed be ‗v‘.

Vertical | **Horizontal**

$u_v = v \sin 48.0^o$
$s_v = 1.20$ m
a = -9.80 ms-2
$t = 7.92 / v$

$s = ut + ½ at^2$
$1.20 = (v \sin 48.0^o)(7.92 / v) + ½ (-9.80) (7.92 / v)^2$
$1.20 = 5.89 – 307 / v^2$
$-4.69 = -307 / v^2$
$v^2 = 65.5$
$\mathbf{v = 8.09 \text{ ms}^{-1}}$

$u_h = v \cos 48.0^o$
$s_h = 5.30$ m
$t = ?$
$u_h = s_h / t$

$t = 5.30 / v \cos 48.0^o$
$\therefore t = 7.92 / v$

20. **Vertical** **Horizontal**

$u_v = 13.0 \text{ Sin } 48.0^0 = 9.66 \text{ ms}^{-1}$
$v_v = 0$
$a = -9.80 \text{ ms}^{-2}$
$s_v = ?$

$v^2 = u^2 + 2as$
$0^2 = 9.66^2 + 2\,(-9.80)\,s$
s_v = 4.76 m < 6.0 m (Does not hit roof)

b) **Vertical**

$u_v = 13.0 \text{ Sin } 48.0^0 = 9.66 \text{ ms}^{-1}$
v_v = -9.66 ms-1 (since s_v = 0 m)
$a = -9.80 \text{ ms}^{-2}$
$t = ?$

$v = u + at$
-9.66 = **9.66** + (-9.80) t
-19.3 = - 9.80 t
t = 1.97 s

Horizontal

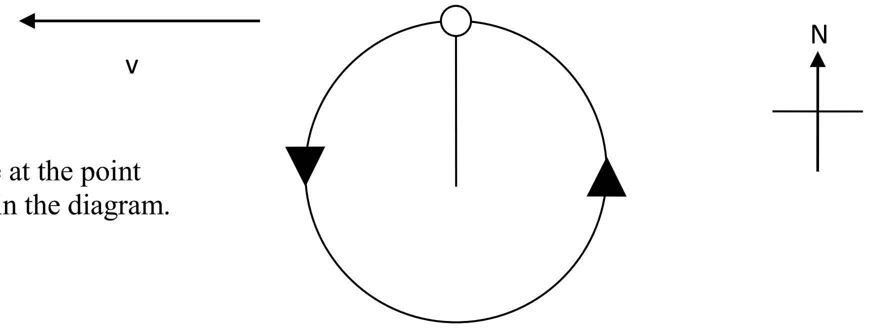
$u_h = 13.0 \text{ Cos } 48.0^0 = 8.70 \text{ ms}^{-1}$
t = 1.97 s
$s_h = ?$

$u_h = s_h / t$
$8.70 = s_h / 1.97$
$s_h = 8.70 \times 1.97$
s_h = 17.1 m < 18.0 m (Lands in court)

21. A long jumper becomes a projectile after they launch themselves. The objective in long jump is to maximise the horizontal displacement achieved by the projectile. Maximising launch speed will help to achieve this objective. The larger the launch speed _v‘ for a particular launch angle (θ), the larger the horizontal component of this velocity (v cos θ) – which is maintained at a constant rate throughout the jump (ignoring friction). The horizontal distance achieved by the long jumper can be represented as v cos θ x t (t = flight time). Given sprinters can achieve a higher value for _v‘ than other runners, they can achieve a longer jump.

Problem Set 4: Circular Motion

1. When the tangential velocity of the metal ball of the hammer is West then the ball should be released. The ball will then, only be influenced by the gravitational force.

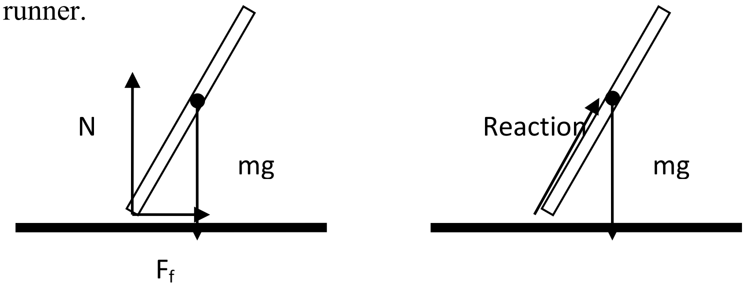


Release at the point shown in the diagram.

2. If you have only completed the projectile motion and circular motion parts of the course then the answer is ……

The sum of the forces on the runner is not equal to zero because the runner is moving in a circle.

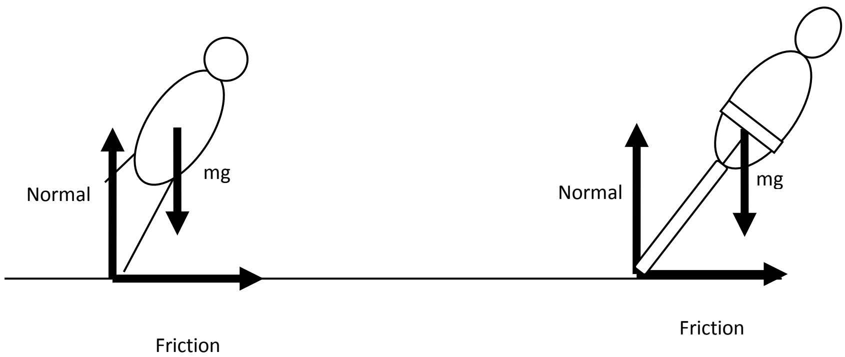
The two forces acting on the runner are mg and the reaction force. The mg force naturally passes through the centre of mass of the object because gravity acts on the centre of mass. The only force that needs to be deliberately placed through the centre of mass therefore is the reaction force. The reaction force consists of two parts or components. They are the normal force and the friction force. The angle formed when these two forces are added is the angle of lean of the runner.



If you have also completed the structures (torque) part of the course the answer is …

The sum of the torques on the runner is equal to zero because the runner is not spinning or toppling. In order for the torques to equal zero all of the forces need to pass through the centre of mass. If both forces (mg and reaction pass through the centre of mass then there will be no radial distance about the pivot placed at the centre of mass. The Normal force counterbalances the weight force (mg). The friction force provides the centripetal force. If the frictional force is applied without the person leaning, the person will topple as their feet take the curve and their centre of mass continues in a straight line at a tangent to the circle.

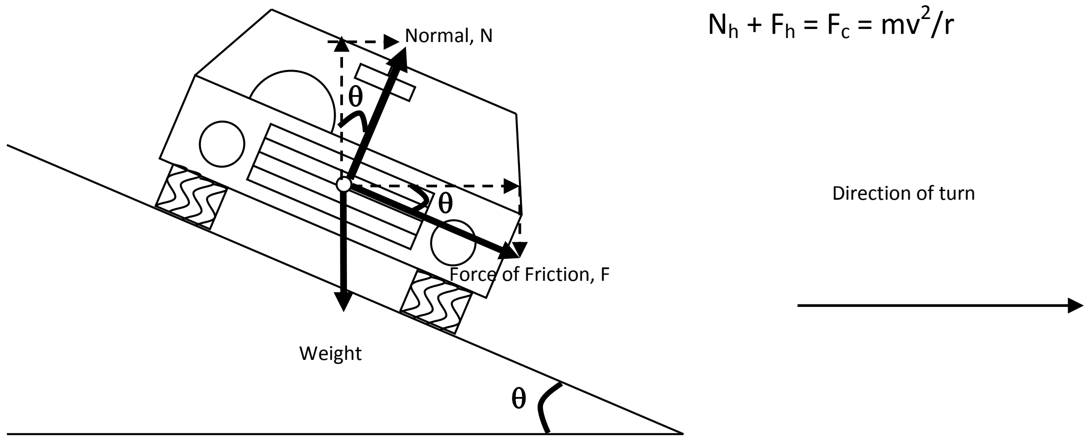
The same principle applies to bicycles rolling around a curve in a flat road.



3. The roller skater travels around the curve because of friction. The roller skater would not be able to round the cure on ice. The skater can also lean towards the turn so as not to solely rely of friction, as described in question 2.

4. Engineers make curves banked so that the normal force can contribute towards the centripetal force. If the curve was not banked, then the tyres would not be able to provided sufficient friction with the road surface to supply the centripetal force necessary to round the curve.

$N_h + F_h = F_c = mv^2/r$



If the angle of banking is θ, then $F_c = N \sin\theta + F \cos\theta = m v^2 / r$

5. $a = v^2 / r$

$a = 3.5^2 / 15$

$a = 0.817$ ms^{-2} towards the centre of the circle.

6. $F_c = m v^2 / r$

$F_c = 0.585 \times 11.5^2 / 1.25$

$F_c = 61.9$ N towards the centre of the circle.

7.a) T = 15.5 s and r = 3.80 m
$v = s / t$

$v = 2\pi r / T$

$v = 2\pi \times 3.8 / 15.5$

$v = 1.54$ ms^{-1}

b) $F_c = m v^2 / r$

$F_c = 28 \times 1.54^2 / 3.8$

$F_c = 17.5$ N towards the centre of the circle.

8. Refer to question 4 for an appropriate diagram

Vertical

$\Sigma F_v = 0$

$N_v + -F_v + -mg = 0$

Let the force of friction = 0

$N \cos\theta = mg$

$N = mg / \cos\theta$ → Sub into horizontal.

Horizontal

$\Sigma F_h = F_c$

$N_h + F_h = mv^2 / r$

Let the force of friction = 0

$N \sin\theta = mv^2 / r$ (insert vertical expression)

$$\frac{mg \sin\theta}{\cos\theta} = \frac{mv^2}{r}$$

$$mg \tan\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{mv^2}{mg\, r}$$

$$\tan\theta = \frac{v^2}{g\, r}$$

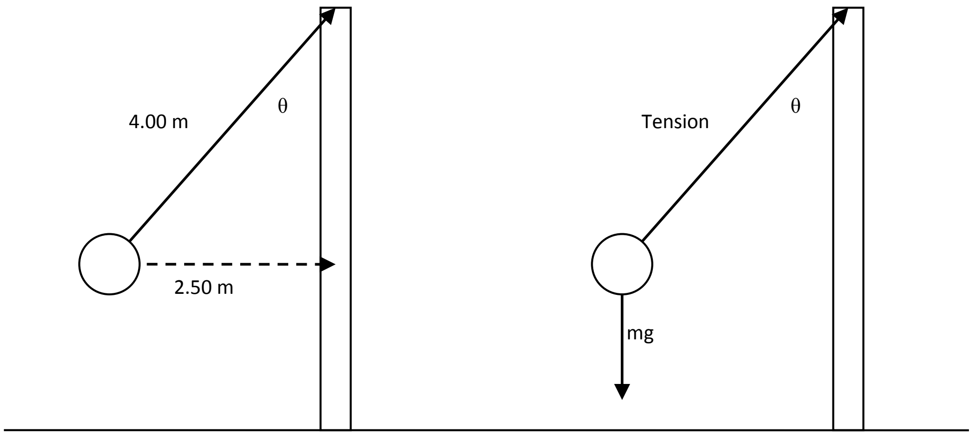
$$\theta = \tan^{-1} \frac{v^2}{g\, r}$$

$$\theta = \tan^{-1} \frac{(110/3.6)^2}{9.8 \times 300}$$

$\boldsymbol{\theta = 17.6^0}$

This result can be used wherever there is no friction on a banked bend. The angle is the angle formed between the ground and the curve – the angle of banking!

9.



a) $\sin\theta = 2.5 / 4$

gives $\theta = 38.7^0$

b) **Vertical** | **Horizontal**

$\Sigma F_v = 0$

$T \cos\theta + -mg = 0$

$T \cos\theta = mg$

$T = 55 \times 9.8 / \cos 38.7$

T = 691 N

$\Sigma F_h = F_c$

$T \sin 38.7 = mv^2 / r$

$691 \sin 38.7 \times 2.5 / 55 = v^2$

$v = 4.43$ m/s

$v = 2\pi r / t$

$t = 2\pi r / v$

$t = 2\pi\, 2.5 / 4.43$

t = 3.55 s

10. a) Yes the car is accelerating because it is **changing direction** (moving in a circle).

b) $F_c = m v^2 / r$
$F_c = 1250 (24/3.6)^2 / 18$
$F_c = 1250 (6.666)^2 / 18$
$F_c = 3.09 \times 10^3$ N (3.09 kN)

c) $\theta = \tan^{-1} \dfrac{(6.6666)^2}{9.8 \times 18}$ (see question 8 for derivation of this equation)

gives $\theta = 14.1^0$

11. $\tan\theta = \dfrac{v^2}{g \times r}$ (see question 8 for derivation of this equation)

$\tan 20 = \dfrac{v^2}{(9.8 \times 70)}$

$9.8 \times 70 \times \tan 20 = v^2 = 249.7\ m^2s^{-2}$
gives $v = 15.8\ ms^{-1}$

12. a) The minimum speed will occur at the shorter radius (9.5 cm)
frequency, f = 3800 RPM = 3800 / 60 = 63.33 Hz
Time period, T = 1 / f therefore f = 1 / T

$v = 2\pi r / T = 2\pi r f$
$v = 2\, \pi\, 9.5\times10^{-2} \times 63.33$

gives $v = 37.8\ ms^{-1}$

b) The maximum centripetal acceleration occurs at the larger radius. If you are unsure your will need to calculate both.

$v = 2\pi r / T = 2\pi rf$

$v = 2\ \pi\ 12.0x10^{-2} \text{ x } 63.33$

$v = 47.76$ m/s

$a = v^2 / r$

$a = 47.75^2 / 12 \text{ x } 10^{-2}$

gives a = $1.90 \text{ x } 10^4$ ms^{-2}

c) The maximum force will be experienced at the $12 \text{ x } 10^{-2}$ m

$F_c = m\ v^2 / r = m(2\pi rf)^2 / r$

$8.2 \text{ x } 10^{-3} = (98 \text{ x } 10^{-9} \text{ x } 10^{-3}) \text{ x } 4\ \pi^2\ 12 \text{ x } 10^{-2} \text{ x } f^2$

$f^2 = 1.77 \text{ x } 10^7$ Hz^2 **gives f = $4.20 \text{ x } 10^3$ Hz**

13.

a) Force is towards the centre of the circle.

b) $F_c = m\ v^2 / r$

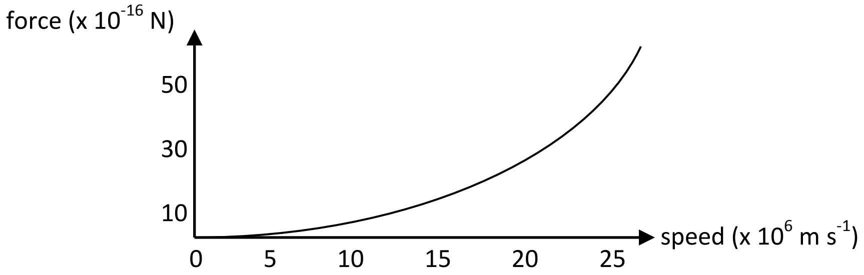
$F_c = 1.7 \text{ x } 10^{-27} \text{ x } (7.8 \text{ x } 10^6)^2 / 200$

$F_c = 5.17 \text{ x } 10^{-16}$ N (if $1.67 \text{ x } 10^{-27}$ kg is used for the mass of a proton, then answer for $F_c = 5.08 \text{ x } 10^{-16}$ N)

c) This is a quadratic relationship, since $F_c = mv^2/r$, therefore F_c □al v^2

So, **doubling** the velocity would require a force **four** times greater to maintain the path. Therefore, if the force is $5.2 \text{ x } 10^{-16}$ N when the velocity is $7.8 \text{ x } 10^6$ m s^{-1}, then the force will be about $21 \text{ x } 10^{-16}$ N when the velocity is doubled (about $16 \text{ x } 10^6$ ms^{-1}) and the force will be about $1.3 \text{ x } 10^{-16}$ N (a quarter of the original force) when the velocity is halved (about $4 \text{ x } 10^6$ ms^{-1}).

The graph would therefore be of the form, $y = x^2$



d) The time period in question should be 2.50 s (and not 2.00 s as quoted in this part of the question. The proton is accelerating and therefore it is completing each revolution quicker and quicker as the time progresses and it is not as simple as multiplying the 440,000 by 0.8 (2.0 ÷ 2.5) to try to obtain the new number of revolutions).

$s = 2\ \pi\ 200 \text{ x } 440\ 000$ = **$5.53 \text{ x } 10^8$ m (553 Mm)**

e) The proton will fall due to the effects of gravity. Assume that the proton has no vertical velocity on entering the synchrotron.

$s = ut + ½ at^2$

$s = 0 + ½ (9.8) \times 2.5^2$

s = 30.6 m down (a significant drop if not taken into account)

f) A field (electric or magnetic) is required to provide an upward force to counterbalance the protons weight.

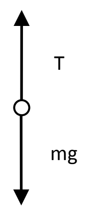
In order to keep the proton circling horizontally, a vertical force must be applied to the proton in order to oppose its weight and hence prevent it losing height.

Opposing force required = weight of proton,
$w = m \times g = 1.7 \times 10^{-27}$ kg x 9.8 ms^{-2} = 1.67×10^{-26} N upwards.
This could be provided by an appropriate magnetic field or electric field.

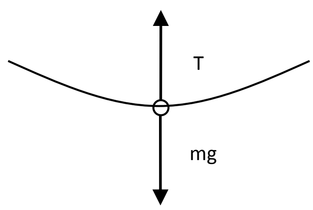
14.

a) The tension in the string provides the centripetal force as well as supporting the objects weight.

Object hanging from string (not swinging)



Object swinging from string



mg + T = 0

T = -mg (forces are equal, but opposite)

$T – mg = mv^2 / r$

$T = mv^2 / r + mg$

Based on the equations above, the tension is greater in the string when the object is swinging because the tension and the centripetal force add and therefore the string is likely to snap.

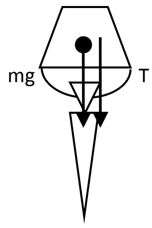
15a) This answer is based on an estimated radius of 0.8 m

The assumption built into the wording of the question is that the –other" force (arm tension of compression) is set to 0.

$\Sigma F_v = mv^2 /r$

$T + mg = mv^2/r$

Let T = 0



$mg = mv^2/r$

$g = v^2/r$

$v = \sqrt{gr}$

$v = \sqrt{(9.8 \times 0.8)}$

$v = 2.8\ ms^{-1}$ (about $3\ ms^{-1}$)

b) The bucket is being ‗driven' towards the centre of its circular path due to the presence of a resultant force providing a centripetal force. The water contents of the bucket, is maintaining its inertia (Newton's First Law) and would ‗feel' like it is being forced towards the outside of the circular path (the misconceived ‗centrifugal' force). Hence, the water remains in the bucket.

This is the same as the effect you feel as a passenger in a car which is turning a corner (much more noticeable if the car is taking the corner at speed!) – the car is being ‗pulled' towards the centre of the turning circle and you feel like you are being pushed outwards, when in reality you are just feeling the car being ‗pulled' inwards.

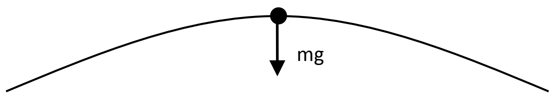
c) The bucket can travel at a constant minimum speed around its circular path, however the arm muscles would have to work a bit harder at different points of its path – the highest and lowest points of its vertical path would be the two extremes. If the arm muscles applied a constant tension force, then the speed would have to be regularly increased and decreased in order to maintain a vertical circular path.

There is also a ‗loss' in potential energy from the top of the path towards the bottom, which would suggest that, since the total amount of energy must remain constant, then there must be a gain in kinetic energy (and therefore speed) in order to compensate for this loss.

16.

a) r = 800m

The other force (reaction force) (lift) on the pilot is 0.



$\Sigma F_v = mv^2 /r$

since lift = 0, then: $mg = mv^2/r$

$mg = mv^2/r$

$g = v^2/r$

$v = \sqrt{gr}$

$v = \sqrt{(9.8 \times 800)}$

$v = 88.5\ ms^{-1}$

b) At the bottom of the loop the plane has lost height, lost E_p (mg□h) and so has gained E_k (½mv^2). The total amount of energy, E_{total} must remain constant, so $E_{total} = E_p + E_k$

Conservation of energy

E_{total} at top = E_{total} at bottom

□h = 2r = 2 x 800 = 1600 m (relative to bottom)

E_k at top + E_p at top = E_k at bottom

½ m u^2 + mg□h = ½ m v^2

½ u^2 + g□h = ½ v^2

½ x 88.5^2 + 9.8 x 1600 = ½ v^2

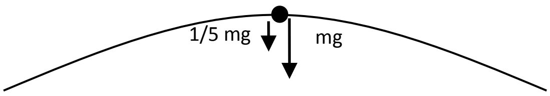
3916 + 15680 = ½ v^2

19596 = ½ v^2

39192 = v^2

gives v = 198 ms^{-1}

17. Two forces act on the pilot, her weight (mg) and the reaction force from her seat (⅕mg) downwards.



$\Sigma F_v = mv^2 /r$

⅕mg + mg = mv^2/r

6/5 mg = mv^2/r

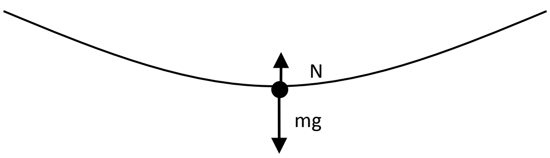
6/5 g = v^2/r

v = √(6/5 g r)

v = √(1.2 x 9.8 x 650)

gives v = 87.4 ms^{-1}

18. a)



$\Sigma F_v = mv^2 /r$

N - mg = mv^2/r

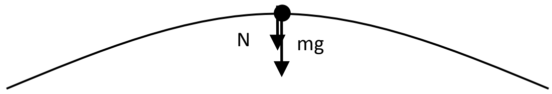
N = mv^2/r + mg

N= (2 x 20^2 / 5) + (2 x 9.8)

N = 160 + 19.6

gives N = 180 N upwards (to 3 sig. figs.)

b)



$$\Sigma F_v = mv^2 / r$$

$N + mg = mv^2/r$

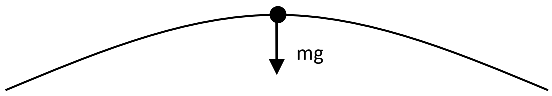
$N = mv^2/r - mg$

$N = (2 \times 10^2 / 5) - (2 \times 9.8)$

$N = 40 - 19.6$

gives N = 20.4 N downwards

19. a)



$$\Sigma F_v = mv^2 / r$$

‗Lift‘ force = 0, since you feel ‗weightless‘

$mg = mv^2/r$

$g = v^2/r$

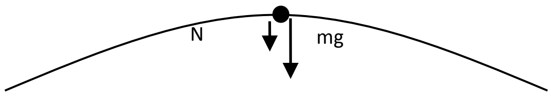
$r = v^2/g$

$r = 14^2 / 9.8$

gives r = 20.0 m

b) If the roller coaster travels faster than 14.0 ms^{-1} more force will need to be supplied downwards to assist the mg force in providing the resultant centripetal force necessary to keep the carriage in the loop. This extra force will come from the normal force (reaction force) of the tracks in a downward direction.

For example, let v = 20 ms^{-1}



$$\Sigma F_v = mv^2 / r$$

$N + mg = mv^2/r$

$N = m (v^2/r - g)$

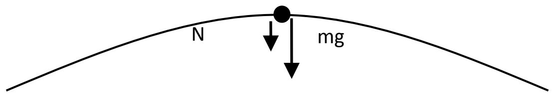
$N = m (20^2/20 - 9.8)$

$N = (20 - 9.8)$ m (i.e. just over 10 times your mass)

N = (10.2 m) N in the direction shown in the free body diagram

c) If the roller coaster travels slower than 14.0 ms^{-1} less force will need to be supplied downwards. A force will need to be provided upwards to reduce the effects of the mg force. This extra force upwards needs to come from the track mechanism, which depending on the track design may actually be physically impossible (if no safety devices).

For example, let v = 10 ms^{-1}



$\Sigma F_v = mv^2/r$

$N + mg = mv^2/r$

$N = mv^2/r - mg$

$N = m(v^2/r - g)$

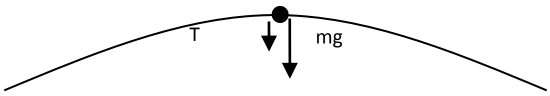
$N = m(10^2/20 - 9.8)$

N = (5 – 9.8) m (i.e. just under 5 times your mass)

N = (-4.8 m) N

N = (4.8 m) N in the <u>opposite</u> direction to that shown in the free body diagram – upwards!

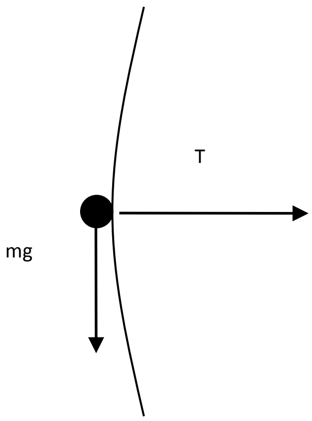
20. a)



where T is the tension force created by her arm muscles / bones / upper body.

b)

Conservation of energy



E_{total} at top = E_{total} at bottom = E_{total} at middle

Δh_{top} = 2 r = 1.8 m (relative to bottom)

Δh_{middle} = r = 0.9 m (when level with bar / relative to bottom)

E_k at top + E_p at top = E_k at middle + E_p at middle

$\frac{1}{2} m u^2 + mg\Delta h_{top} = \frac{1}{2} m v^2 + mg\Delta h_{middle}$

$\frac{1}{2} u^2 + g\Delta h_{top} = \frac{1}{2} v^2 + g\Delta h_{middle}$

$\frac{1}{2} \times 1^2 + 9.8 \times 1.8 = \frac{1}{2} v^2 + 9.8 \times 0.9$

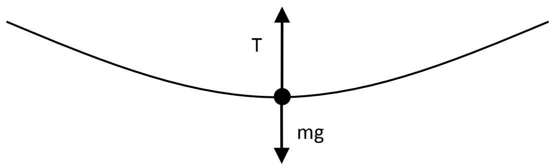
$\frac{1}{2} + 17.64 = \frac{1}{2} v^2 + 8.82$

$18.14 - 8.82 = 9.32 = \frac{1}{2} v^2$

$18.64 = v^2$

gives v = 4.32 ms^{-1} directly downwards

c)



Conservation of energy

E_{total} at top = E_{total} at bottom

$\Delta h_{top} = 2\ r = 1.8$ m (relative to bottom)

E_k at top + E_p at top = E_k at bottom
(since relative E_p at bottom = 0)

$\frac{1}{2}\ m\ u^2 + mg\Delta h_{top} = \frac{1}{2}\ m\ v^2$

$\frac{1}{2}\ u^2 + g\Delta h_{top} = \frac{1}{2}\ v^2$

$\frac{1}{2} \times 1^2 + 9.8 \times 1.8 = \frac{1}{2}\ v^2$

$\frac{1}{2} + 17.64 = \frac{1}{2}\ v^2$

$18.14 = \frac{1}{2}\ v^2 \quad \therefore \quad 36.28 = v^2$

gives v = 6.02 ms^{-1}

$\Sigma F_v = mv^2/r$

$T - mg = mv^2/r$

$T = mv^2/r + mg$

$T = (40 \times 6.02^2 / 0.9) + (40 \times 9.8)$

$T = 1611 + 392$

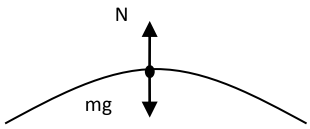
T = 2003 N upwards

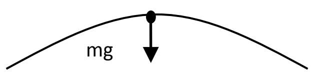
(2.00 kN upwards, provided by a tension force created by her arm muscles / bones / upper body)

21. a)

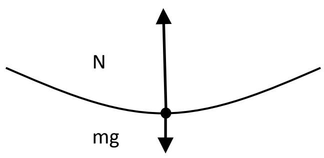
LOW — HIGH

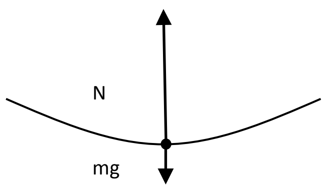
Top





Bottom





b) Based on feeling ‗weightless' on the HIGH setting, the speed, v of the Ferris wheel is: (refer to diagram ‗top right')

$mg = mv^2 / r$

$g = v^2 / r$

$v = \sqrt{(r\ g)}$

$v = \sqrt{(3.6 \times 9.8)}$

gives $v = 5.94\ ms^{-1}$

c) On the **HIGH** setting:

At the top, reaction force = 0 due to the feeling of 'weightlessness'

However, at the bottom (refer to diagram ‗bottom right'):

$N - mg = mv^2 / r \therefore\ N = mv^2 / r + mg$ (passengers revolve at constant speed)

$N = (60 \times 5.94^2 / 3.6) + (60 \times 9.8) \therefore\ N = 588 + 588$

gives reaction force, N = 1176 N (1.18 kN)

d) On the **LOW** setting (speed is half the HIGH speed = 5.94 / 2 = 2.97 ms^{-1}):

At the top (refer to diagram ‗top left'):
$mg - N = mv^2 / r$
$N = mg - mv^2 / r$
$N = (60 \times 9.8) - (60 \times 2.97^2 / 3.6) \therefore\ N = 588 - 147$
gives reaction force, N = 441 N

At the bottom (refer to diagram ‗bottom left'):

$N – mg = mv^2 / r$
$N = mv^2 / r + mg$
$N = (60 \times 2.97^2 / 3.6) + (60 \times 9.8) \therefore\ N = 588 + 147$
gives reaction force, N = 735 N

22.

a) E_{total} at top = E_{total} at bottom = E_{total} at middle

□$h_{point\ A} = 2\ r = 4.00$ m (relative to bottom of swing)

□$h_{point\ X} = r = 2.00$ m (relative to bottom of swing)

□$h_{point\ B} = 0$ (relative to bottom of swing)

Consider the situation of the stone at Point A (highest point of its swing):

E_k at point A + E_p at point A = E_k at point X + E_p at point X

$\frac{1}{2}\ m\ u_A^2 + mg□h_{point\ A} = \frac{1}{2}\ m\ V_X^2 + mg□h_{point\ X}$

$\frac{1}{2}\ u_A^2 + g□h_{point\ A} = \frac{1}{2}\ v^2 + g□h_{point\ X}$

$\frac{1}{2} \times u_A^2 + (9.8 \times 4) = \frac{1}{2} \times 10.4^2 + (9.8 \times 2)$

$\frac{1}{2} \times u_A^2 + 39.2 = 54.08 + 19.6 = 73.68$

$73.68 - 39.2 = 34.48 = \frac{1}{2} \times u_A^2$

$68.96 = u_A^2$

gives $u_A = 8.30\ ms^{-1}$

Now, consider the situation of the stone at Point B (lowest point of its swing):

E_k at point A + E_p at point A = E_k at point B + E_p at point B

$\frac{1}{2}\ m\ u_A^2 + mg□h_{point\ A} = \frac{1}{2}\ m\ V_B^2 + mg□h_{point\ B}$

$\frac{1}{2}\ u_A^2 + g□h_{point\ A} = \frac{1}{2}\ V_B^2 + g□h_{point\ B} + 0$

$\frac{1}{2} \times 8.3^2 + (9.8 \times 4) = \frac{1}{2}\ V_B^2$

$34.45 + 39.2 = \frac{1}{2}\ V_B^2$

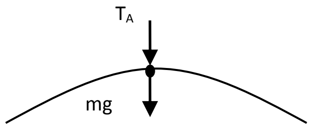
$73.65 = \frac{1}{2}\ V_B^2$

$147.3 = V_B^2$

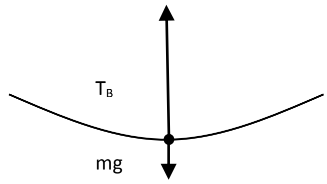
gives $V_B = 12.1\ ms^{-1}$

22. b)

Top of swing (i.e. at point A)



Bottom of swing (i.e. at point B)



Tension at the top = T_A

$mg + T_A = mv^2 / r$

$T_A = mv^2 /r - mg$

$T_A = (2.5 \times 8.3^2 / 2) - (2.5 \times 9.8)$

$T_A = 86.11 – 24.5$

gives T_A = 61.6 N downwards

Tension at the bottom = T_B

$T_B - mg = mv^2 / r$

$T_B = mv^2 /r + mg$

$T_B = (2.5 \times 12.1^2 / 2) + (2.5 \times 9.8)$

$T_B = 183 + 24.5 = 207.5$

gives T_B = 208 N upwards (3 sig. figs.)

c) The string is most likely to break **at the bottom** (**point B**), since the tension is greater here. The tension must counterbalance weight <u>and</u> also provide the centripetal force. Mathematical verification:

Tension at top (point A), T_A	**Tension when stone is horizontal (at point X), T_X**	**Tension at bottom (point B), T_B**
	$T_X = mV_X^2 / r = (2.5 \times 10.4^2 / 2)$	
61.6 N	(since ‗mg‘ has no effect horizontally	**208 N**
(as given above)	**gives T_X = 135 N**	(as given above)

Set 5: Gravitation and Satellites

1. You are attracted to large buildings. Unfortunately the mass of yourself and the large building is not large enough to create a measureable force.

   For example, assume that a 100 kg man is standing 1 m from a building of mass 10,000 tonnes (10 million kg).

   The force of attraction between the building and this man, $F = G\, M_{building}\, M_{man} / r^2$

   $F = 6.67 \times 10^{-11} \times 1 \times 10^{7} \times 100 / 1^2 = 6.67 \times 10^{-2}$ N

   (friction between the ground and feet of the man would be significantly greater)

2. a) Yes their weight does change but not by much. The increase in radial distance from the centre of the planet is not large enough to produce a measurable effect or an effect that is noticeable to the brain of the climber.

   Since $g = GM_{earth} / r^2$ then $g\ \alpha^{al}\ 1/r^2$ (where r = distance from centre of Earth)

   On Earth‘s surface, $r = 6.37 \times 10^{6}$ m

   On Mount Everest, $r = (6.37 \times 10^{6} + 8000) = 6.378 \times 10^{6}$ m, therefore their weight would **decrease** by a very small amount due to this slightly increased distance.

   b) The weight of an underground miner decreases as they descend into a mine, though the effect is not easily measured. The reason that this occurs is that as you descend into the Earth, the Earth above you attracts you upwards slightly while the Earth below you continues to pull you down, though to a slightly reduced extent. If you were able to descend to the centre of the Earth you would eventually become weightless because you would have equal quantities of matter all around you, pulling you equally in all directions. The universal gravitational law does not operate below the surface of the Earth. Instead the gravitational field drops to zero linearly as you move from the surface of the Earth to its centre.

   c) The density of the Earth is not uniform. More dense rocks (rocks that have more mass per unit of volume they take up) in the crust will give a stronger gravitational reading than lighter rocks. Also, the Earth is not perfectly spherical and its radius differs at different locations around its surface, leading to higher readings of the acceleration due to gravity where the radius is smaller.

3. The object has weight (mg). The object appears ―weightless” as it begins to fall because there are no other forces acting on it. The perception (feeling) of weight is due the presence of other forces such as normal force or air resistance opposing the weight force. The falling object will continue to ―feel” weightless until it approaches terminal velocity and the air resistance force becomes appreciable. The air resistance force will then provide the ―other” force that will allow you to ―feel” your weight.

4. The acceleration of an object is not determined by the mass of the object. It is determined by the mass of the planet (other objects) that is making (generating) the gravitational field.

   In the formula for gravitational filed strength created by a planet, $g = GM_p / r^2$ the mass of the object is not listed and therefore irrelevant. Only the mass of the planet and the distance from the centre of the planet (field), are listed.

5. The formula for the force, F acting on an object in the gravitational field of a star is:

$F = G M_{object} M_{star} / r^2$. This formula can only be applied outside of the star's surface. Inside the star, a different formula holds true. If the volume of the star is reduced however without altering the stars mass, the edge of the star becomes closer to the centre of the star and the formula above will still hold true for longer until you pass below the star's surface.

Decreasing the volume of an object without altering its mass increases the density of the object ($\rho = M / V$ rearranged, $M = \rho\uparrow V\downarrow$).

In the formula $F = G M_{object} M_{star} / r^2$, as r decreases, the size of the force (and consequently gravitational field) increases. If r is small enough without going inside the stars surface, the strength of gravitational field will be so great that not even light can escape.

6. $g = G M / r^2$

$10 = 6.67 \times 10^{-11} \times M_{earth} / (6.37 \times 10^6)^2$

$M = 6.08 \times 10^{24}$ kg

7. $F = G M_1 M_2 / r^2$

$F = 6.67 \times 10^{-11} \times 100 \times 100 / 0.622^2$

$F = 1.724 \times 10^{-6}$ N

8. a)

On surface

$F = G M_{earth} M_{shuttle} / r^2$ ------- (1)

Above surface

$\frac{1}{2} F = G M_{earth} M_{shuttle} / X^2$ ---------- (2)

Substitute F on surface (1) into F above surface (2)

$$\frac{\frac{1}{2} G M_{earth} M_{shuttle}}{r^2} = \frac{G M_{earth} M_{shuttle}}{X^2}$$

Cancelling common terms gives:

$$\frac{\frac{1}{2}}{r^2} = \frac{1}{X^2}$$

therefore $\frac{1}{2} X^2 = r^2$ so $X^2 = 2 r^2$ and **$X = \sqrt{2}r$**

if $r = 6.37 \times 10^6$ m, then $X = 9.01 \times 10^6$ m

Height above the surface will be $(X - R_{earth}) = (9.01 \times 10^7 - 6.37 \times 10^6) = h$

gives **$h = 2.64 \times 10^6$ m**

8. b) $g = G M_{earth} / r^2$

$g = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / (6.37 \times 10^6 + 610 \times 10^3)^2$

$g = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / (6.98 \times 10^6)^2$

$g = 8.17$ m/s^2 towards the earth

$F_{gravitaitonal} = F_{centripetal}$

$$\frac{G M_{earth} M_{telescope}}{r^2} = \frac{M_{telescope} v^2}{r}$$

$$\frac{G M_{earth}}{r} = v^2$$

c) $v^2 = \dfrac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.98 \times 10^6} = 5.70 \times 10^7 \text{ m}^2\text{s}^{-2}$

$v = 7.55 \times 10^3$ m/s (7.55 kms^{-1})

9. $F = G M_{earth} M_{moon} / r^2$

$2.03 \times 10^{20} = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22} / r^2$

$r^2 = 1.44 \times 10^{17}$ m^2

$r = 3.80 \times 10^8$ m

10.

g on surface of Earth

$g_e = G M_e / r_e^2$

g on surface of Neptune

$$g_n = \frac{G\ 16.6M_e}{(3.89r_e)^2}$$

$$g_n = \frac{G M_e \times 16.6}{(r_e \times 3.89)^2}$$

$$g_n = \frac{G M_e \times 16.6}{r_e^2 \times 15.1321}$$

$$g_n = \frac{G M_e \times 1.097}{r_e^{\ 2}}$$

But $g_e = G M_e / r_e^2$ substituting in we get

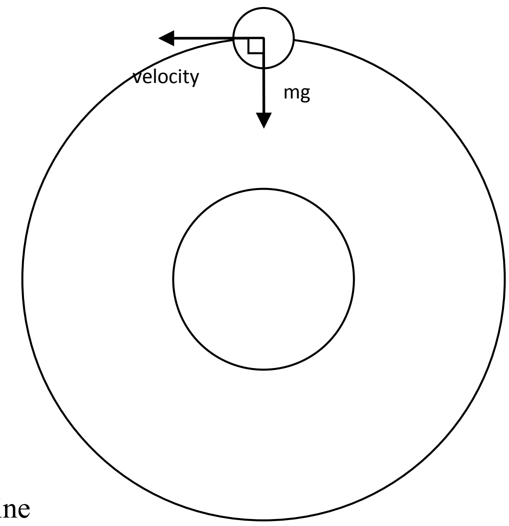
$g_n = g_e \times 1.097$

The ratio $g_e : g_n$ is 1:1.10

(If $g_e = 9.80$ then $g_n = 9.80 \times 1.097 =$ **10.8 m/s^2**)

11. a) The gravitational force of the earth acts at right angles to the velocity of the moon. The acceleration of the earth‘s gravity acting on the mass of the moon (moons weight) has no component that is in the same direction as the velocity of the moon and so cannot assist it to alter its velocity.

> Note :- This is not a free body diagram because free body diagrams **only** list the forces acting on the object. Velocity is not a force.



Also:

$$F_{gravitaitonal} = F_{centripetal}$$

$$\frac{G\,M_{earth}\,M_{moon}}{r^2} = \frac{M_{moon}\,v^2}{r}$$

$$\frac{G\,M_{earth}}{r} = v^2$$

Therefore only the Earth‘s mass and the distance the moon is from the Earth determine the moon‘s speed; since both are constant, then the moon‘s speed is unchanged.

b) During a solar eclipse the Earth, moon and sun are all in a line. The total force on the moon will be the vector sum of the force effects of the earth and the sun.

F of Earth on Moon

$F_e = G\,M_{earth}\,M_{moon} / r_{em}^2$

$$F_e = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}}{(3.8 \times 10^8)^2}$$

$F_e = 2.027 \times 10^{20}$ N

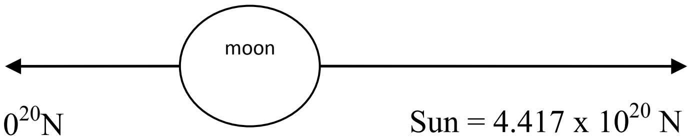
F of Sun on Moon

$F_s = G\,M_{sun}\,M_{moon} / r_{sm}^2$

$$F_s = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 7.35 \times 10^{22}}{(1.49 \times 10^{11} - 3.8 \times 10^8)^2}$$

$$F_s = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 7.35 \times 10^{22}}{(1.4862 \times 10^{11})^2}$$

$F_s = 4.417 \times 10^{20}$ N



Earth = 2.027×10^{20}N   Sun = 4.417×10^{20} N

Net Force = $4.417 \times 10^{20} - 2.027 \times 10^{20}$

Net Force = 2.39×10^{20} N towards the sun

12. The greater the distance the longer the period.

According to Kepler's law $\frac{r^3}{T^2} = \frac{G\,m_p}{4\,\pi^2}$ as r gets bigger T gets bigger also since $\frac{G\,m_p}{4\,\pi^2}$ is constant. So, $T^2\ \alpha^{al}\ r^3$, therefore a satellite orbiting at a height in excess of 30,000 km will have the greater time period.

13. a) The satellite's orbit would change from circular to elliptical. The balance between the speed of the satellite and the centripetal force has been broken. If a constant retarding force is encountered such (as the Earth's atmosphere), the pathway taken by the satellite will become an inward spiral (death spiral).

b) The satellite's period of revolution is matched to the period of the Earth's rotation. This is achieved using Kepler's law $\frac{r^3}{T^2} = \frac{G\,m_p}{4\,\pi^2}$ to calculate the correct distance above the earth to park the satellite in order to achieve an orbital period equal to 24 hours. The satellite does not fall back to earth because its rate of acceleration towards the Earth, is counterbalanced by its rate movement (velocity) away from the Earth at a tangent.

14. a) To move a satellite into orbit it is necessary to increase the gravitational potential energy of the satellite. This is done by giving the satellite kinetic energy, usually provided by a rocket. The calculation according to the law of conservation of energy begins to look like:

E_p (at Earth's radius, r_{earth}) + E_k (rocket) = E_p (at r_{earth} + altitude)

An object on the equator already has some kinetic energy due to the revolution of objects on the Earth's surface as compared to an object at a geographic pole (just rotating). This means a satellite on the equator needs less supplementary (extra) kinetic energy to get it into orbit. This is why the USA's space agency is in Florida on the equator.

The Earth rotates towards the East. If the rocket is shot in the opposite direction to the rotation of the Earth (west) you are actually removing the kinetic energy that the Earth has given to the satellite. If the rocket is shot in the same direction to the rotation of the Earth (west) you are adding to the kinetic energy that the Earth has given to the satellite, almost like a catapult effect. This is what you want.

b) See 14. a) above.
The speed of rotation of the earth is at a maximum at the equator and so the kinetic energy given to the rocket / satellite by the Earth's rotation is also at a maximum. It will require less rocket fuel to make up the extra kinetic energy required to get the satellite into orbit.

15. Reduce the orbiting speed of the capsule so the capsule goes into an elliptical orbit, bringing it in contact with the Earth's atmosphere resulting in an inward spiral to Earth.

16. According to the law of conservation of energy, the gravitational potential energy is being converted into kinetic energy as the satellite descends.

When you use the more general form of the gravitational potential energy, including the fact that it drops off with added distance from the Earth, $V = -G\frac{M_{earth} M_{satellite}}{r}$,

then the choice of zero potential is different. In this case we generally choose the zero of gravitational potential energy at infinity, since the gravitational force approaches zero at infinity, hence the reason why the above expression is negative. This is a sensible way to define the 'zero point' since the potential energy with respect to a point at infinity tells us the energy with which an object is fixed to the Earth. So, by decreasing its kinetic energy, its potential energy increases (becoming less negative) and can only do so when the radial distance, r decreases, thus forcing the satellite into a lower orbit. Once in this lower orbit, its speed $v = \sqrt{(G M_{earth} / r)}$
(see solution to question 8, part c).), therefore a reduced radius of orbit will result in an increase in the satellite's speed.

17. Gravity is still present. They still have a weight. The astronauts and space capsule are falling towards the earth however, resulting in the free fall ―weightless" effect. When an object is in freefall, it experiences no Normal (other) force. ―Weightlessness" is the absence of any other force to counterbalance the weight force. The seatbelt however provides an external resultant force which keeps the astronauts in position.

18. Kepler's law suggests that: $r^3 / T^2 = GM_{earth} / 4\pi^2$

$r^3 \times 4\pi^2 / GM_{earth} = T^2$ (assuming a polar orbit at an altitude of 550 km), then:

$T^2 = (5.50 \times 10^5 + 6.37 \times 10^6)^3 \times 4\pi^2/(6.67 \times 10^{-11} \times 5.97 \times 10^{24}) = 3.285 \times 10^7\ s^2$

$T = 5.73 \times 10^3$ s (1.59 hours)

19. a) Europa's orbital speed, $v = 2\pi r / T$

$v = 2\ \pi\ 6.71 \times 10^8 / 3.07 \times 10^5$

$v = 1.37 \times 10^4\ ms^{-1}$ ($13.7\ kms^{-1}$)

b) Jupiter's mass, M_J, given by:

$r^3 / T^2 = G M_J / 4\pi^2$

$(6.71 \times 10^8)^3 / (3.07 \times 10^5)^2 = 6.67 \times 10^{-11} \times M_J / 4\pi^2$

$M_J = 1.90 \times 10^{27}$ kg

20. $r^3 / T^2 = G M_{earth} / 4\pi^2$

$(2.02 \times 10^7 + 6.37 \times 10^6)^3 / (12 \times 3600)^2 = 6.67 \times 10^{-11} \times M_{earth} / 4\pi^2$

$(2.657 \times 10^7)^3 / (43200)^2 = 6.67 \times 10^{-11} \times M_{earth} / 4\pi^2$

$M_{earth} = 5.95 \times 10^{24}$ kg

21. The satellite is geostationary. It has an orbital period of 24 hours (86400 s).

$r^3 / T^2 = G M_{earth} / 4\pi^2$, therefore: $r^3 / (24 \times 3600)^2 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / 4\pi^2$

so, $r^3 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times (86400)^2 / 4\pi^2$ gives $r^3 = 7.5295 \times 10^{22}$

therefore $r = 4.223 \times 10^7$ m, suggesting that height above Earth, $h = 3.59 \times 10^7$ m

22.

Radius of Moon - Earth orbit

$r_e{}^3 = G M_{earth} \times (27.3 \times 24 \times 3600)^2 / 4\pi^2$

Radius of Titan - Saturn orbit

$r_s{}^3 = G M_{saturn} T^2 / 4\pi^2$

$r_s{}^3 = G\ 108 M_{earth} \times (14 \times 24 \times 3600)^{22} / 4\pi^2$

Form ratio (fraction):

$$\frac{r_e{}^3}{r_s{}^3} = \frac{G M_{earth} \times (27.3 \times 24 \times 3600)^2 / 4\pi^2}{G\ 108 M_{earth} \times (14 \times 24 \times 3600)^2 / 4\pi^2}$$

$$\frac{r_e{}^3}{r_s{}^3} = \frac{(27.3)^2}{108 \times (14)^2}$$

$$\frac{r_e{}^3}{r_s{}^3} = 0.035208$$

take cube root of both sides

$r_e = 0.3278\ r_s$ → $r_s = r_e/0.3277$ → $r_s = 3.05\ r_e$

$r_{\text{titan's orbit around Saturn}} = 3.05\ r_{\text{moon's orbit around Earth}}$ The ratio $r_{titan} : r_{moon}$ is 1:3.05

23. a) Without being provided with the mass of Mercury, only the **relative** force of attraction can be calculated:

Closest approach (perihelion)

$R_{peri} = 4.60 \times 10^{10}$ m

$F_{peri} = G M_{sun} M_{mercury} / R_{peri}{}^2$

Furthest approach (aphelion)

$R_{aphe} = 6.90 \times 10^{10}$ m

$F_{aphe} = G M_{sun} M_{mercury} / R_{aphe}{}^2$

Dividing both expressions cancels all the constant factors, leaving:

$$F_{peri} / F_{aphe} = R_{aphe}^2 / R_{peri}^2 = (6.9 \times 10^{10})^2 / (4.6 \times 10^{10})^2 = 2.25$$

so the attractive force at Mercury's perihelion due to the Sun is 2.25 times greater than the force of attraction at its aphelion

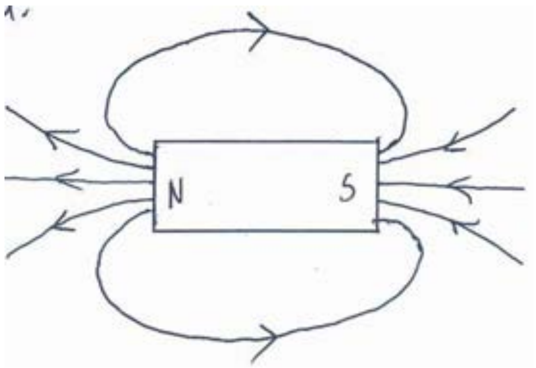
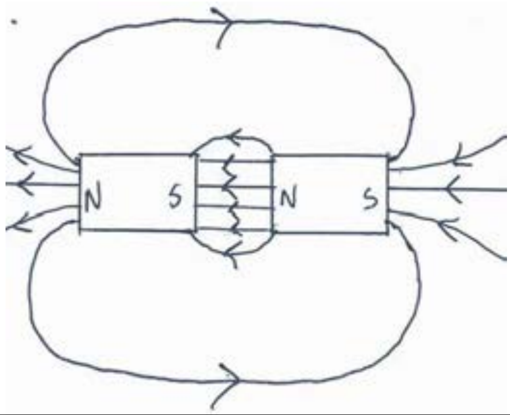
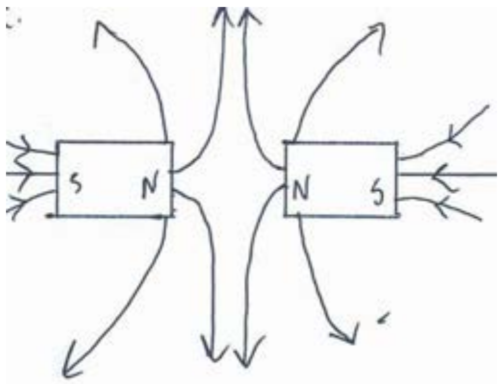
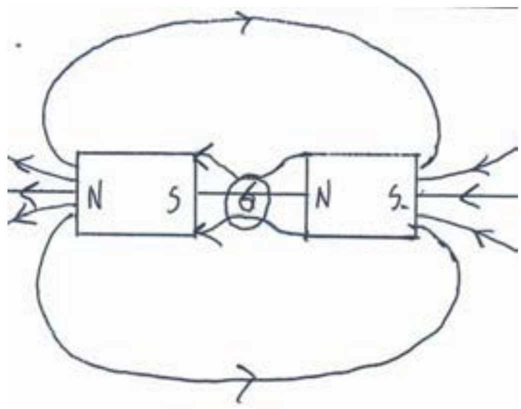
23. b) The respective velocities will also be different:

Closest approach (perihelion)	Furthest approach (aphelion)
$R_{peri} = 4.60 \times 10^{10}$ m	$R_{aphe} = 6.90 \times 10^{10}$ m
$v^2 = \frac{GM_{sun}}{R_{peri}}$	$v^2 = \frac{GM_{sun}}{R_{aphe}}$
$v^2 = \frac{6.67 \times 10^{-11}\ 1.99 \times 10^{30}}{4.60 \times 10^{10}}$	$v^2 = \frac{6.67 \times 10^{-11}\ 1.99 \times 10^{30}}{6.90 \times 10^{10}}$
$v^2 = 2.886 \times 10^9\ m^2s^{-2}$	$v^2 = 1.924 \times 10^9\ m^2s^{-2}$
$v = 5.37 \times 10^4\ ms^{-1}$ ($53.7\ kms^{-1}$)	**$v = 4.39 \times 10^4\ ms^{-1}$ ($43.9\ kms^{-1}$)**

c) Conservation of Energy. The energy possessed by Mercury is either in a gravitational potential form or in a translational kinetic energy form. None of its energy is lost to other objects (no energy transfer). If its total orbital energy remains constant then it will not decay (inward spiral) into the sun.

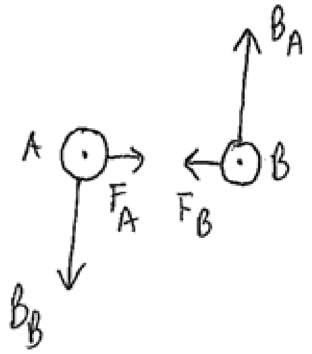
Problem Set 6: Magnetic fields and forces

1.

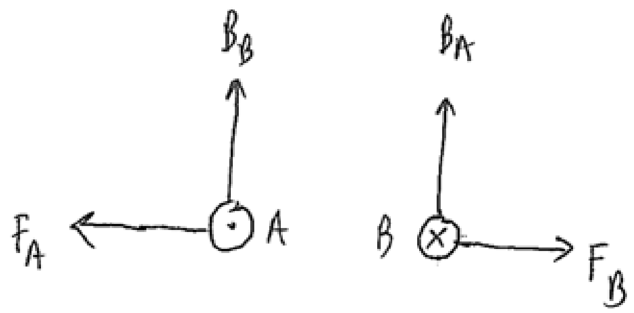
a	b
	
c	**d**
	

2. The presence of the ferromagnetic materials in the surroundings will alter the direction of the magnetic field lines. This is because ferromagnetic objects contain magnetic domains, making them more permeable to magnetic flux. This results in magnetic flux lines changing direction in order to pass through iron objects instead of passing through air or a vacuum.

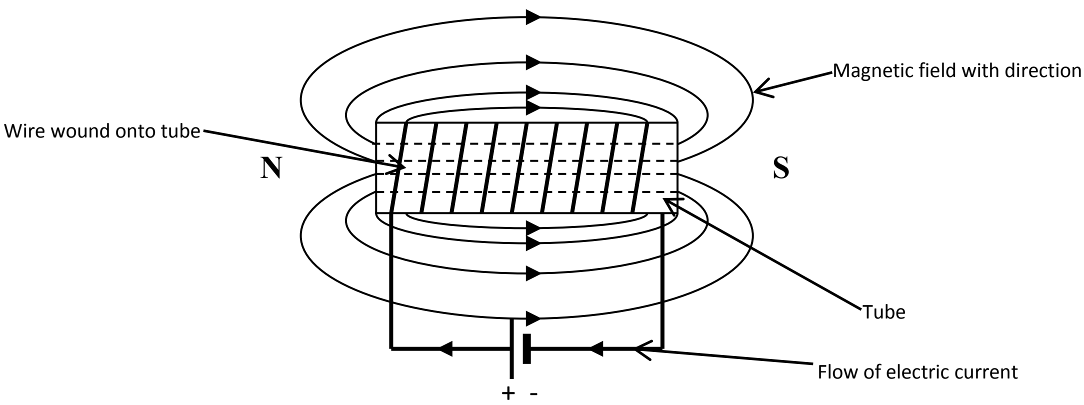
3. a) If both wires carry current in the same direction they will experience a force of attraction towards each other.



b) If the wires carry current in the opposite direction to each other then they will experience a force of repulsion away from each other.



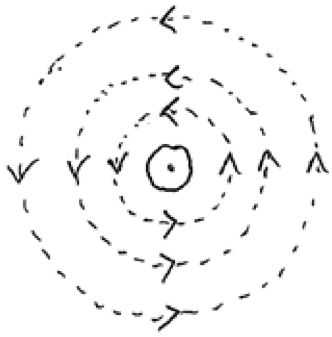
4.



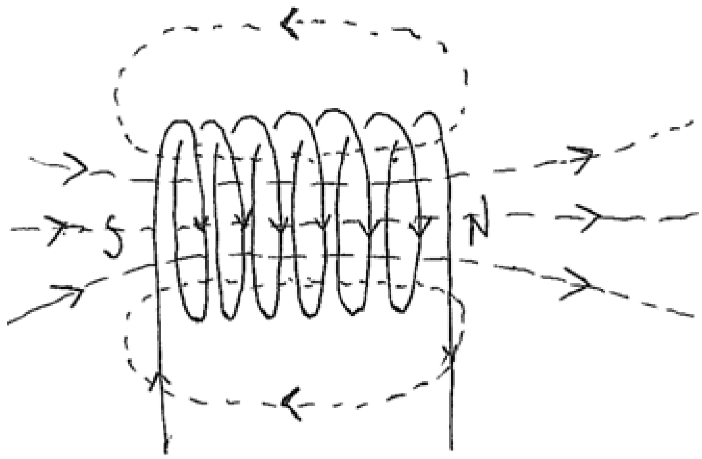
5. Assume the whole of the side length is contained in the field.

$\Sigma\tau = 2 \times Fr$
$= IBlnw$
$= 0.200 \times 0.350 \times 0.1 \times 200 \times 0.0300$
$\mathbf{= 4.20 \times 10^{-2}\ N\ m}$

6.a) A current carrying wire



b) A circular coil



7. Electrical power cords produce magnetic fields when a current is flowing through them. The information is stored on the tapes as specific alignments of magnetic domains in the tape‟s material. The cords‟ magnetic fields could alter the magnetic information stored on the tape by causing the domains on the magnetic tape to realign. This can result in the loss of the information.

8. a) Towards the top of the page.
   b) Towards the top of the page.
   c) Towards the top of the page.
   d) No resultant force – charges are moving parallel with the field.

9. a) When the doorbell‟s switch is closed, the current-carrying solenoid produces a magnetic field. This field magnetises the soft iron rod which is, therefore, attracted to the bell (which must be made of a ferromagnetic material). This force of attraction causes the rod to accelerate to the left. Even though the iron rod‟s acceleration drops to zero when it is in the centre of the solenoid, its momentum continues its motion towards the left and, eventually, the rod strikes the bell.

   b) The bell will ring louder if the iron rod strikes it with a greater force. The attractive force acting on the piece of soft iron can be increased by:
   - increasing the number of turns in the solenoid (increases „B‟)
   - increasing the amount of current that is flowing in the wire (also increases „B‟).

   (there may be others)

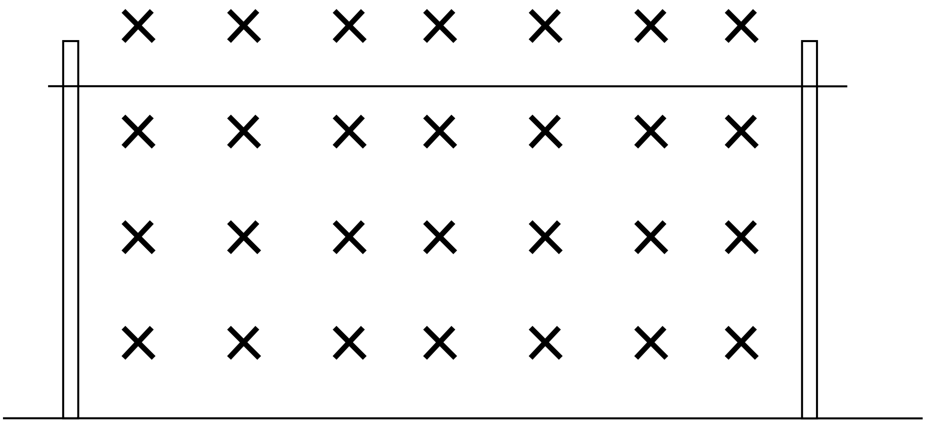
10. Assume n = 1:
    F = nIlB
    F = 1 x 10.0 x 0.12 x 2.00
    F = 2.40 N
11. The windings (armatures) experience the most torque when they are parallel to the magnetic field (the magnetic force experienced by the coil is at its maximum distance from the pivot of the coil in this position).

This torque reduces to zero as a winding rotates to a perpendicular position relative to the field (r = 0). In other words, the torque experienced by a single winding varies according to a sinusoidal function as it rotates – from a maximum to zero, and so on.

The 12 windings in a commercial motor are arranged at angles to each other within a 360º arc. A winding is only connected to the DC power supply when it is parallel to the magnetic field and experiencing maximum torque. Hence, at any given time, the motor is mostly experiencing maximum torque.

This means the torque produced by the motor remains relatively constant instead of varying between a maximum and zero torque as would be the case for a single armature.

12.



Viewed from the south looking north. X represents Earth external magnetic field.

F= IlB

$F = 40 \times 75 \times 5 \times 10^{-5}$

F = 0.150 N

The direction of the force depends on the direction of the current:

- If conventional current flows to the west, then the magnetic force acts towards the bottom of the page.
- If conventional current flows to the east, then the magnetic force acts towards the top of the page.

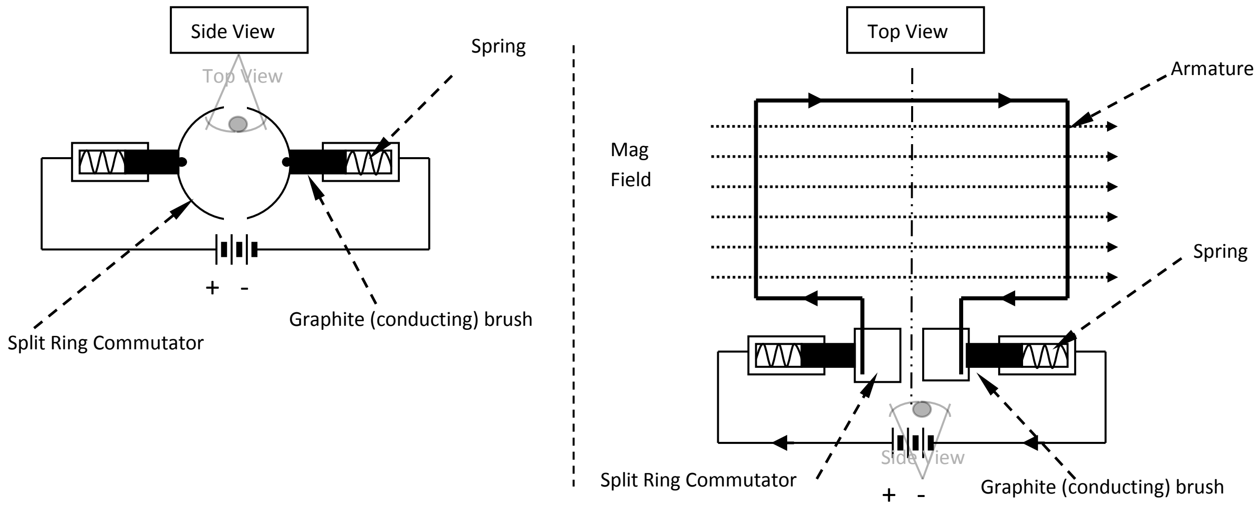
13. a) Out of the page.
    b) Due to Newton"s third law, into the page.
    c) The rails is in a fixed positon on the earth, the train is not. Therefore, because the magnet is attached to the train, the train will move into the page.
    d) Armature, magnetic field and commutator.
    e) The construction of each is very similar. There is, however, a difference in the inputs and outputs.

	Input	Construction	Output
Generator	Motion	Slip rings	Electricity
Motor	Electricity	Commutator	Motion

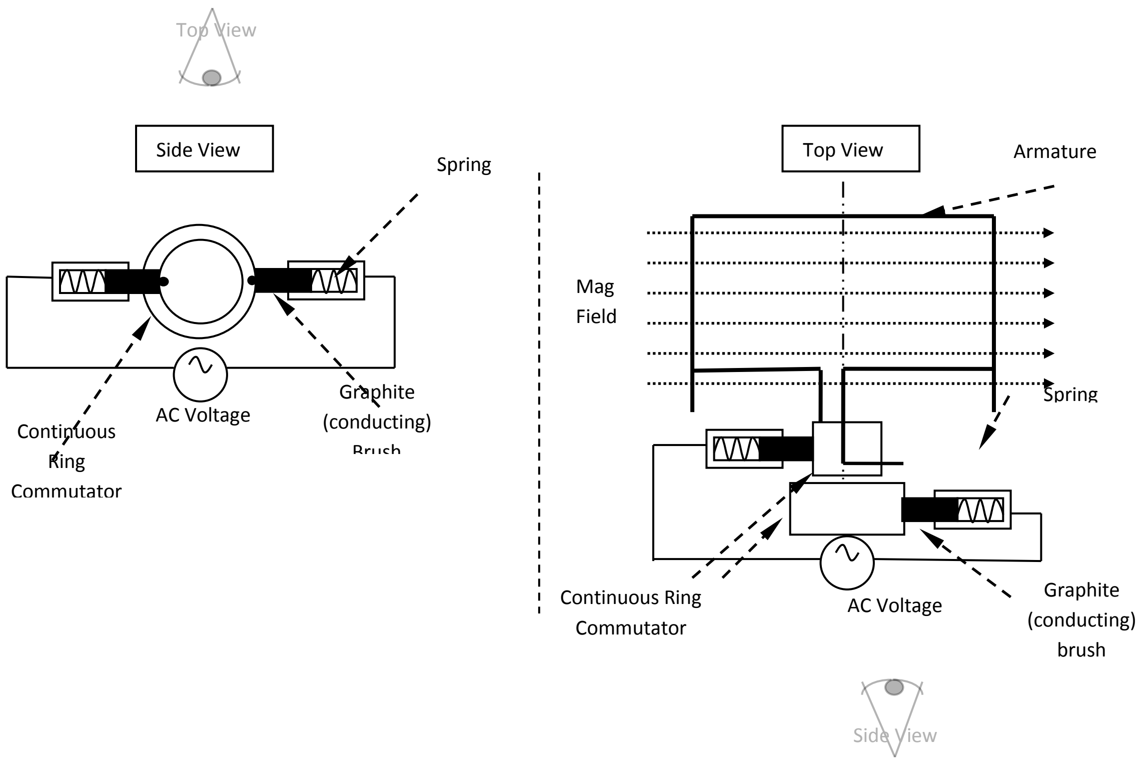
f) Applying the brakes would switch off the electric current that is supplied to the rail. The movement of the magnet past the charges in the rail induces an emf and an electric current in the rail. According to Lenz"s Law, the direction of this induced emf and current will oppose the change that produced it (ie – the motion of the train). Hence, the train slows down.

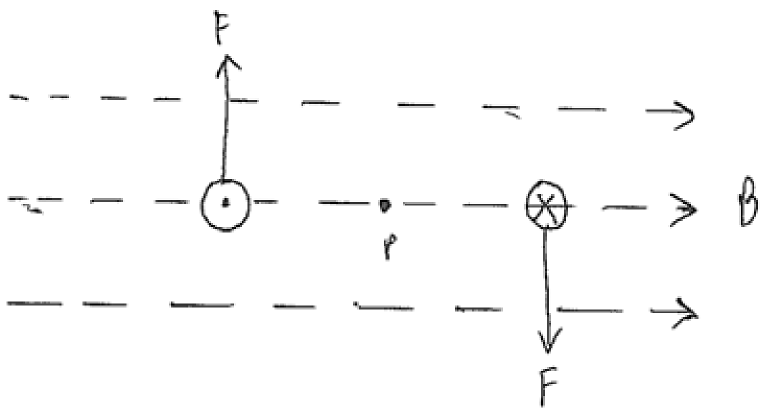
14.

Split Ring Commutator - Direct Current Motor



Continuous Ring Commutator - Alternating Current Motor





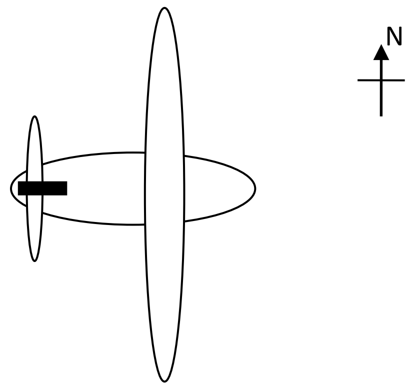
Problem Set 7: Magnetic Induction

1. emf = nlvB

v = 980 km h^{-1} × 1000/3600 = 272.22 m s^{-1}

emf = 1 × 60.0 × (980 / 3.6) × 3.50 × 10^{-5}

emf = 0.572 V



2. a) % efficiency = —— × 100

90 = $\frac{50}{P_{in}}$ × 100

P_{in} = ———

P_{in} = 55.6 W

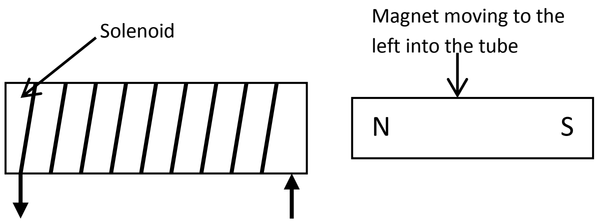
P = VI

55.56 = 240 × I

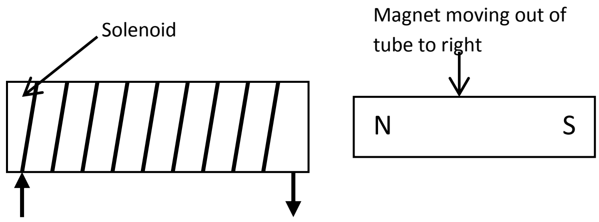
I = 0.231 A

b) The television set is not 100 % efficient. Much of the energy that is lost is converted to heat. This heat must be vented to the surrounding or it will cause the internal components of the television to over heat.

3. Lenz‟s Law. When the magnet is inserted a current is induced by the increasing field strength. The current is induced in such a direction as to oppose the field that created it. This means that the current that is induced creates its own magnetic field that repels (opposes) the increasing magnetic field strength.



When the magnet is withdrawn a current is induced in the opposite direction to that above. This is because the external magnetic field is weakening. The current induced creates a magnetic field that attempts to attract the external magnet back.



4\. a) When the primary circuit closes, a current flows. The change from no current to a steady current, induces a magnetic field for that very short period of time. That magnetic field induces a current in the secondary. Since turning the switch on is a very quick change, the current noted in the secondary coil will only be induced for a very short time hence "transitory".

b) Increase the voltage supplied to the primary.
Decrease the switching time (difficult!).
Increase the ratio of the coils in the secondary to that in the primary.
Link the primary and secondary coils using a hoop of soft iron.

c) Applying Lenz's law, the current in the secondary is opposite that of the current in the primary when the switch is closed and the current increases. As the current is shown travelling from left to right through the primary coil in the diagram, the current through the secondary coil will be right to left.

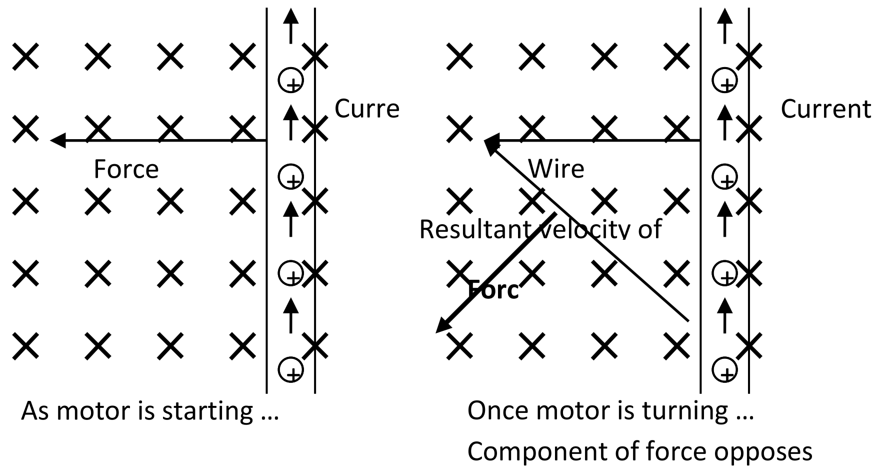
5\. a) By moving in an easterly direction, the aerial will be cutting across the field at 90^0 and so an EMF will be induced.

EMF = nlvB
v = 60.0 km h^{-1} = 60.0 x 1000/3600 = 16.67 m s^{-1}
EMF = 1 x 0.500 x 16.67 x 2.50 x 10^{-5}
EMF = 2.08 x 10^{-4} V or 0.208 mV

b) Same answer as part a. The alternative unit is **2.08 x 10^{-4} Wb s^{-1}**

c) The current will change. The aerial is no longer cutting across flux lines but running parallel to the Earth's magnetic field so no current will be induced.

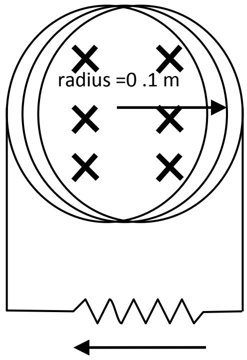
6\. Lenz's law. Current induces a force. The force induces a current in opposite direction. This current opposes the original current and shows up as an increased resistance on the coil as the speed increases. This is also known as a back-emf. If as the motor spins the current induced was in the same direction as the original current, too much current would flow and this would result in the motor burning out. This would violate the law of conservation of energy.



7\. emf = n Δ (BA) / t = n (BA – BA) / t
area of a coil = π r^2
emf = 200 (0.5 x π x 0.015^2 – 0) / 10
emf = 20 (0.5 x π 0.015^2 – 0) / 1

emf = 7.07×10^{-3} V

8\. a) emf = n (BA – BA) / t
emf = 1 (0.25 x π 0.1^2 – 0) / 0.2
emf = 3.92×10^{-2} V



b), c)
Based on this diagram the direction of the current is right to left through the resistor.

d) V = IR
3.92×10^{-2} = I x 5
I = 7.84×10^{-3} A

9\. a) emf = nlvB
v = 80 km h^{-1} x 1000/3600 = 22.2 m s^{-1}
emf = 1 x 1.00 x 22.2 x 36.0 x 10^{-6}
emf = 7.99×10^{-4} V

b) If the earth's magnetic field is out of the earth the force on the electrons will be south east. If the earth's magnetic field is into the earth the force on the electrons will be north west.

10 . Lenz's law. As the plate approaches the magnet the field strength increases. This induces a current (eddy current) in the plate. The eddy current creates its own magnetic field in opposition to the strengthening original external magnetic field. This causes repulsion and slows the approach of the plate.

As the plate passes out the other side the external magnetic field experienced by the plate is decreasing in strength. This induces a current in the plate. The current creates its own magnetic field in support of the weakening external magnetic field. This attracts the plate and slows its movement away from the magnet.

The slowing on approach and slowing on exit causes the swing of the plate to be damped and so the pendulum comes to a stop sooner.

11\. Assuming that 180 V is the average voltage (RMS)
T = 1 / 50
T = 0.02 s
Change in flux will happen over a quarter of a turn so t = ¼ T = 0.005s

emf = nΔBA / t
emf = n (BA – BA) / t
180 = n (0.200 x 2.00 x 10^{-2} – 0) / 0.005
n = 225 turns

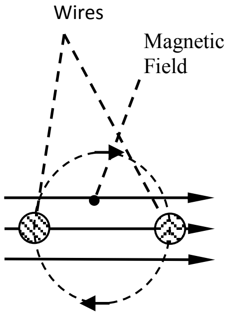
12.a) l = 20.0 mm = 0.0200 m
IR = nlvB
10.0 x 10^{-3} x 2.50 = 1 x 0.0200 x v x 0.500
v = 2.50 m s^{-1}

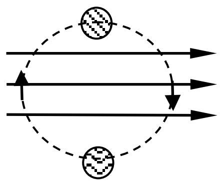
b) F = IlB
F = 10.0 x 10^{-3} x 0.0200 x 0.500
F = 1.00×10^{-4} N

13.a) T = 1 / 60.0
T = 0.0167 s
t = ¼ T **= 0.00417 s**

emf = n (BA – BA) / t
240 = 300 (B x 0.2 x 0.2 – 0) / 0.00417 s
B = 8.33 x 10^{-2} T

b)





Maximum Voltage
Wires cut across flux

Minimum Voltage
Wires move parallel with flux

14. The metal will usually be of a ferromagnetic type. The current through the coil magnetises the iron, and the field of the magnetised material will add to the field produced by the coil. The core can increase the magnetic field of a coil by many times over what it would be without the core, resulting in a larger deflection for any given current.

15. Need to make an assumption regarding the direction the plane is flying in respect to the Earth‟s magnetic field.

Assuming the plane‟s wing is at right angles to the Earth‟s magnetic field, then the EMF induced will be at a maximum and

From question:

l = 64.0

v = 920 km h^{-1} x 1000/3600 = 255.6 m s^{-1}

B = 1.02 x 10^{-5} T

Induced EMF, ε = l v B   ε = 64.0 x 255.6 x 1.02 x 10^{-5} = **0.167 V**

16. a)

$$e = N\frac{(f_2 - f_1)}{t}\sin q$$

$$e = N´\ \frac{(B_2A - B_1A)}{t}\sin q$$

$$e = N´\ \frac{(B_2 - B_1)A}{t}\sin q$$

Area of the coil, A = Πr^2 = 3.1416 x 0.0800^2 = 20.11 x 10^{-3} m^2

B_2 = 3.95

$B_1 = 0.850$

N= 45.0

$\boldsymbol{\theta} = 300\text{-}270 = 30^0$

$$e = 45.0 \times \frac{(3.95 - 0.850) \times 20.11 \times 10^{-3}}{450 \times 10^{-3}} \times \sin 30$$

$$e = 3.117 = 3.12V$$

b) V = IR or I = V/R

R = 12.8 Ω

$$I = \frac{V}{R}$$

$$I = \frac{3.117}{12.8} = 0.2435 = 0.244A$$

17.

$$e = N \times \frac{\Delta(BA_{\perp})}{t}$$

Area of the coil, A = Πr^2

Diameter of the coil = 6.80 cm = 0.0680 m and r = 0.0680/2 = 0.0340

A = 3.1416 x 0.0340^2 = 3.632 x 10^{-3} m^2

ΔB = 250 mT = 250 x 10^{-3} T

N = 60.0 turns

t = 3.50 s; and so

$$e = 60.0 \times \frac{(250 \times 10^{-3} \times 3.632 \times 10^{-3})}{3.50}$$

$$e = 15.6 \times 10^{-3}V$$

18. a)

$$e = N \times \frac{\Delta(BA_{\perp})}{t}$$

Area of the coil, A = l x w = 0.0500 x 0.0180 = 900 x 10^{-6} m^2

N = 300 turns

B = 0.180 T

t = ¼ T = ¼ x 1/f = ¼ x 1/60 = 4.167 x 10^{-3} s

$$e = 300 \times \frac{(0.180 \times 900 \times 10^{-6})}{4.167 \times 10^{-3}}$$

$$e = 11.663 = 11.7V$$

b)

$$e_{rms} = \frac{e_{max}}{\sqrt{2}} = \frac{11.663}{\sqrt{2}} = 8.247 = 8.25V$$

c) Curved poles keeps more of the coil more perpendicular to the magnetic field for a greater amount of time.

19.

$$e = N \times \frac{\Delta(BA_{\perp})}{t}$$

N = 85.0 turns

B = 0.250 T

A = 3.10 x 10^{-2} m^2

t = ¼ T = ¼ x 1/f = ¼ x 1/3600 = 69.44 x 10^{-6} s

$$e = 85.0 \times \frac{(0.250 \times 3.10 \times 10^{-2})}{69.44 \times 10^{-6}}$$

$$e = 9487 = 9.49kV$$

$$e_{rms} = \frac{e_{max}}{\sqrt{2}} = \frac{9487}{\sqrt{2}} = 6708 = 6.70kV$$

20.

$$e = N \times \frac{\Delta(BA_{\perp})}{t}$$

N= 240 turns

B = 0.860 T

Area of the coil, A = Πr^2

Diameter of the coil = 12.0 cm = 0.120 m and r = 0.120/2 = 0.060

A = 3.1416 x 0.060^2 = 11.310 x 10^{-3} m^2

t = ¼ T = ¼ x 1/f = ¼ x 1/2400 = 104.17 x 10^{-6} s

$$e = 240 \times \frac{(0.860 \times 11.310 \times 10^{-3})}{104.17 \times 10^{-6}}$$

$$e = 22.409 \times 10^{3} = 22.4kV$$

$$e_{rms} = \frac{e_{max}}{\sqrt{2}} = \frac{22.409 \times 10^{3}}{\sqrt{2}} = 15.846 \times 10^{3} = 15.8kV$$

21.

$$e = N \times \frac{\Delta(BA_{\perp})}{t}$$

N= 1500 turns

B = 51.0 μ T = 51.0 x 10^{-6} T

Area of the coil, A = Πr^2

Diameter of the coil = 0.240 m and r = 0.240/2 = 0.120

A = 3.1416 x 0.120^2 = 0.04524 m^2

t = 1/2 T since the coil is rotated through only 180^0 = 1/2 x 2.50 x 10^{-3} = 1.25 x 10^{-3} s

$$e = 1500 \times \frac{(51.0 \times 10^{-6} \times 0.04524)}{1.25 \times 10^{-3}}$$

$$e = 2.769 = 2.77V$$

22.

$$\varepsilon = N \times \frac{\Delta(BA_{\perp})}{t}$$

Rearranging for N

$$N = \frac{\varepsilon \times t}{\Delta(BA_{\perp})}$$

ε = 240 RMS = 240 x √2 = 339.41 V peak

Diameter of the coil = 7.60 cm = 0.0760 and r = 0.0760/2 = 0.0380

A = 3.1416 x 0.0380^2 = 0.00454 m^2

B = 300 mT = 300 x 10^{-3} T

t = ¼ T = ¼ x 1/f = ¼ x 1/50 = 0.005 s

$$N = \frac{339.41 \times 0.005}{(300 \times 10^{-3} \times 0.00454)}$$

$$N = 1246\,turns, say\,1250\,turns$$

23. Note incorrect wording in question. Coils are rectangular hence can not have a diameter – assume 6.00 cm long, 8.00 cm *wide*

$$\varepsilon = N \times \frac{\Delta(BA_{\perp})}{t}$$

Rearranging for B

$$B = \frac{\varepsilon \times t}{NA}$$

ε = 20.0 V peak

N = 400 turns

Area of each coil = 0.0600 x 0.0800 = 0.00480 m^2

t = ¼ T = ¼ x 1/f

f = 400 rpm = 400 / 60 = 6.667 Hz

so t = 0.0375 s

$$B = \frac{20.0 \times 0.0375}{400 \times 0.00480}$$

$$B = 0.391\,T$$

Problem Set 8: Electrical energy and power

1a) $P = V \times I$
$P = 3000 \times 20$
$P = 60\,000$ W = 60.0 kW

b) $E = P \times t$
$E = 60\,000 \times 5 \times 10^{-3}$
$E = 300$ J

c) $V = IR$
$3000 = 20 \times R$
$R = \frac{3000}{20}$
$R = 150\ \Omega$

2a) $P_{total} = P_{per\ meter\ on\ panel} \text{ x } 10/100 \times n_{metres\ square}$
$2000 = 1373 \text{ x } 0.1 \times n_{metres\ square}$
$n = 14.6\ m^2$

b) $P = V^2 / R$
$2000 = 50^2 / R$
$R = 1.25\ \Omega$

3a) $P = V \text{ x } I$
$125\,000 = 1500 \times I$
$I = 83.3333$ A = 83.3 A

b) $V = IR$
$1500 = 83.3333 \times R$
$R = 18.0\ \Omega$

4 The thicker wire is on the low voltage side of the transformer. In a step down transformer this is on the secondary side. If the power out of the transformer is similar to the power in, then based on P = VI, if the voltage drops on the secondary side then the current must increase. This increase of current flow results in a heating effect in the secondary wire due to the resistance of the wire. If the wire is made thicker then the resistance of the wire is reduced based on the resistivity formula
$R = \rho l / A$

5 Current in the outer coil is anti-clockwise and increasing at a constant rate. By Lenz‟s Law the current on the inner coil is clockwise and steady.
Primary current created a strengthening field. Strengthening field induces current in the secondary. The magnetic field created by the secondary current is in such a direction as to oppose the change that created it.

6a) $\frac{V_p}{V_s} = \frac{I_s}{I_p}$

$\frac{240}{12\,000} = \frac{I_s}{I_p}$

$\frac{I_s}{I_p} = \frac{240}{12\,000}$

$I_s = I_p$
1 : 50
0.02 : 1 times

b) $\frac{V_p}{V_s} = \frac{N_p}{N_s}$

$\frac{240}{12\,000} = \frac{200}{x}$

N_s = 10 000 turns

c) 98% of your answer to part a). No power is quoted.
Ratio = 0.02 × 98/100 = 0.0196 : 1

7a) $P = V^2 / R$
$2500 = 240^2 / R$
$R = = 240^2 / 2500$
R = 23.0 Ω

b) V = I R
240 = I × 23
I = 10.43 A = 10.4 A

I = q / t
q = **10.43 × 2 × 60**
q = 1.25 × 10^3 C

8a) We assume that the energy is lost before arriving in the motor.

% efficiency = $\frac{\text{out}}{\text{in}}$ × 100

80 = $\frac{1000}{\text{in}}$ × 100

in = 100 000 / 80

in = 1250 W

P = VI
1250 = 12 × I

I = 104.17 A = 104 A

b) The wires connecting the battery to the starter motor are thicker than the other wires. This is because the current supplied to the starter motor is large Since $P_{loss} = I^2 R_{wire}$ when the resistance in the wire is large, the power loss will be large. By using a thicker wire, the resistance of the wire is decreased and hence the power losses are decreased.

9a) An electricity sub-station is a transformer converting the higher voltages from direct supply from the power station to lower voltage in preparation for distributing out to the local area. Power from the power station is supplied at high voltages to reduce the supply current and hence power losses. Typical voltage drop is from 250 kV initially to 66kV or 33 kV and then to 340 V

b) i) Train systems operate at higher voltages than neighbourhood houses.
ii) The power demand of a train is near zero when stationary, very large when accelerating and medium when running at a constant velocity. These fluctuations in demand will result in fluctuations in the amount of power available to houses also connected to the substation.

c) The voltage is stepped up in order to minimise the current flowing in the high tension (high voltage) wires. By decreasing the current, it decreases the power loss in the transmission line.
$P_{loss} = I^2R_{lines}$

10a) Assume that the transformers are 100% efficient, power losses are only in the transmission lines.

$P = VI$
$500\,000 = 2000 \times I$
$\mathbf{I = 250\ A}$

$P_{loss} = I^2 R$
$P_{loss} = 250^2 \times 0.500$
$\mathbf{P_{loss} = 31\,250\ W = 31.2\ kW}$

b) $P = VI$
$500\,000 = 20000 \times I$
$\mathbf{I = 25.0\ A}$

$P_{loss} = I^2 R$
$P_{loss} = 25^2 \times 0.500$
$\mathbf{P_{loss} = 312.5\ W = 312\ W}$

c) Increase voltage, decrease current, decease power loss by $P_{loss} = I^2R_{lines}$.

11 Transmisson pylons have to be high enough above the ground to reduce the effect of the magnetic field induced by the power lines. These would otherwise induce currents in metallic objects in the surroundings (houses, cars etc). Eventually the cost of the larger pylons outweighs the cost associated with the power loss.

12a) $P_{loss\ in\ cable} = V_{loss\ in\ cable} \times I$
$P_{loss\ in\ cable} = (415 - 405) \times 200$
$P_{loss\ in\ cable} = 10 \times 200$
$\mathbf{P_{loss\ in\ cable} = 2000\ W}$

b) $P_{loss\ in\ cable} = I^2 R$
$2000 = 200^2 R$
$\mathbf{R = 0.05\ \Omega}$

R = length × Resistance per metre
0.05 = length × 0.40
Length = 0.05 / 0.40 = 0.125 m

c) The power loss can be reduced by reducing the current in the wire. Changing the transformer ratios to increase output voltage will decrease the output current and allow the motor to work at a larger distance.

13a) $\%\ efficiency = \frac{out}{in} \times 100$

$80 = \frac{5000}{in} \times 100$

in = 500 000 / 80 **input from motor = 6250 W = 6.25 kW**

b) Heat losses in converting the chemical potential energy in the fuel to mechanical energy in the generator, heat energy losses due to friction in bearings, resistance in the windings of the armature. Sound.

14a) The voltage available to other devices should remain at 240 V. If a lot of current is supplied to one device, for example the electric motors or arc welding equipment, the mains may difficulty supplying sufficient power so some dimming of lights or reduction in speed of the motor may occur. (brown out)

b) As more power is supplied to a house an increasing amount of current has to flow in the supply lines to the house. This will cause heating of the supply lines according to the power loss formula ($P_{loss} = I^2R_{lines}$)

c) If excessive current is drawn by one device, insufficient power may be available for the other appliances running including the lights. The lights may dim.

15 Train runs away from the transformer along the lines until the voltage available = 20.0 kV

Current provided by transformer.
$P = VI$
$3 \times 10^6 = 25\,000 \times I$
$I = 120$ A

Voltage drop along line is from 25.0 kV to 20.0 kV.

Voltage drop = 25.0 kV – 20.0 kV = 5 kV = 5 000 V.

$P_{loss} = V \times I$
$P_{loss} = 5000 \times 120$
$P_{loss} = 600\,000$ W = 600 kW

$P_{loss} = I^2 R$ 600 000 W
600 000 = $120^2 \times R$
$R = 41.67\ \Omega$

R_{line} = (R per km) × (distance in km)
41.67 = 1.2 × (distance in km)
(distance in km) = 34.7 km or approximately 35 km.

16. a) There are four main causes of inefficiency in transformers. List any two:

Resistance of the windings

Flux leakage – i.e. poor design may mean not all of the flux from the primary coil is linked to the secondary coil.

Eddy currents in the iron core

Hysteresis – Constant reversing of the magnetic field in the core expends energy.

All but flux leakage lead to energy losses due to heating.

b) V_p = 240 V rms

V_s = ?

N_p = 300 turns

N_s = 4800 turns

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$V_s = \frac{V_p \times N_s}{N_p} = \frac{240 \times 4800}{300} = 3840 Vrms$$

$$V_s peak = V_s rms \times \sqrt{2} \times \% efficiency$$

$$V_s peak = 3840 \times \sqrt{2} \times \frac{92}{100} = 4996 = 5.00 kV$$

c) $I_s$peak = $I_s$rms x √2= 40 x 10^{-3} x √2= 0.05657 A

P_s peak = 4996 x 0.05657 = 282.62 W

Based on efficiency losses P_p peak = P_s peak x 100/92 = 307.20 W

$V_p$peak = $V_p$rms x √2= 240 x √2 = 339.41

$$P = VI$$

$$I = \frac{P}{V} = \frac{307.20}{339.41} = 0.905 A$$

17. $V_p = 240$ V

Step down: $V_s = 6.30$ V, $I_s = 8.00$ A, $P_s = VI = 6.30 \times 8.00 = 50.4$ W, $N_s = 84.0$ turns

Step up: $V_s = 35000$ V, $I_s = 15.0 \times 10^{-3}$ A, $P_s = VI = 35000 \times 15.0 \times 10^{-3} = 525$ W

a)

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$N_p = \frac{V_p \times N_s}{V_s}$$

$$N_p = \frac{240 \times 84.0}{6.30} = 3200\,turns$$

b)

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$N_s = \frac{V_s \times N_p}{V_p}$$

$$N_s = \frac{35000 \times 3200}{240} = 466667\,turns$$

c) Total power drawn in the primary coil = sum of the secondaries

$P_p = 50.4 + 525 = 575.4$ W

$$P = VI$$

$$I = \frac{P}{V}$$

$$I = \frac{575.4}{240} = 2.3975 = 2.40\,A$$

d) The coils in the step up transformer would use considerably finer wire than the step down. As the current is comparably smaller, power losses are minimised. The step down transformer should have thicker wire in order to minimise resistive power losses with a higher current.

18.a) Back emf ε = I R = 9.00 x 20.0 = 180 V

Operating voltage across the coil = V – ε = 414 – 180 = 234 V

b) At switch on, the motor is not turning so there is no back emf

$$R_T = \frac{V}{I} = \frac{414}{12} = 34.5\,\Omega$$
$$R_s = R_T - R_m$$
$$R_s = 34.5 - 20.0 = 14.5\,\Omega$$

19.a) Initially the motor is not turning so there is no back emf, and

$$I = \frac{V}{R} = \frac{240}{6.30} = 38.095 = 38.1A$$

b) At full speed, the back emf opposes the exterior source, and

$$V - \varepsilon = IR$$
$$240 - 212 = I \times 6.30$$
$$I = \frac{240 - 212}{6.30} = 4.44A$$

20.

At start up, there is no back emf so

$$R_{motor} = \frac{V}{I} = \frac{12}{5.00} = 2.40\,\Omega$$

At full speed, there is a back emf and

$$V - \varepsilon = IR$$
$$\varepsilon = V - IR$$
$$\varepsilon = 12 - 1.20 \times 2.40 = 9.12V$$

21. Initially, the motor is not turning and there is no induced back emf. The current is very high. As the motor spins up to operating speed, the back emf increases and the current being drawn reduces.

Problem Set 9: Charged particles in an electric field

1. $F = q\,E$

   $7.20 \times 10^{-13} = 1.60 \times 10^{-19} \times \mathbf{E}$

   $\mathbf{E = 4.50 \times 10^{6}\ N\ C^{-1}}$

2. By the square rule …

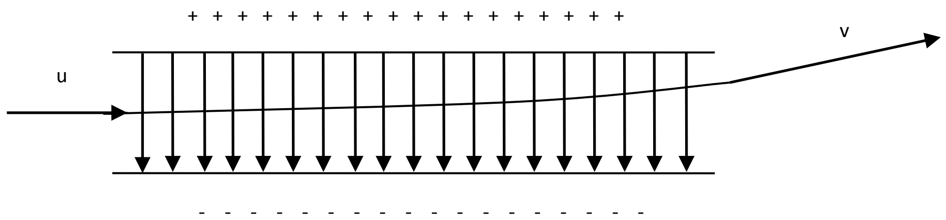
   $F = q \times E$ so $E = F / q = E$, hence E is measured in N/C

   By the square rule …

   $V = E\,s$ so $E = V / s$, hence E is measured in V / m.

   As both can be used to calculate E then the units are equivalent.

3a)



b) initial velocity (u) is purely to the right

Final velocity (v) is the same to the right but also contains a component towards the top of the page. This component has been provided by the electric field.

Time for which the electric field acts = the time for which the charge is between the plates.

$v = s / t$

$t = s / v$

$t = 0.03 / 2.9 \times 10^{7}$

$\mathbf{t = 1.0344 \times 10^{-9}\ s}$

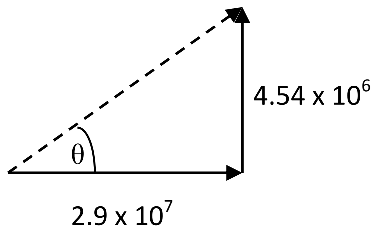
The force provided by the field causes the electron to accelerate towards the positive plate.

$qE = m\,(v - u) / t$

$1.6 \text{ x } 10^{-19} \times 2.5 \times 10^{4} = 9.11 \times 10^{-31}\ (v - 0) / 1.0344 \times 10^{-9}$

$\mathbf{v = 4.54 \times 10^{6}\ m/s}$ **parallel to the field**

Combine the horizontal and the vertical velocities to find the angle.



Magnitude

$R = \sqrt{([2.9 \times 10^{7}]^{2} + [4.54 \times 10^{6}]^{2}}$

$\mathbf{R = 2.94 \times 10^{7}\ m/s}$

Angle

$\tan\theta = 4.54 \times 10^{6} / 2.9 \times 10^{7}$

$\mathbf{\theta = 8.90^{0}}$

Answer $\mathbf{= 2.94 \times 10^{7}\ m/s}$ **at** $\mathbf{8.90^{0}}$ **to the original direction.**

3c) The electron experiences a force which causes an acceleration towards the positive plate (i.e. towards the top of the page as shown in the diagram).

d) Electrons are deflected toward a particular point on the screen. The electrons on striking the screen cause a chemical on the screen to fluoresce. This creates an image on the screen.

e) $W = qV$
$W = 1.6 \times 10^{-19} \times 1.80 \times 10^{3}$
$\mathbf{W = 2.88 \times 10^{-16}\ J}$

f) $V = E\ s$
$E = V / s$

$E = 1.80 \times 10^{3} / 30.0 \times 10^{-3}$
$\mathbf{E = 6.00 \times 10^{4}\ N\ C^{-1}}$

4. This is done to remove the interference of external (unwanted) radio signals and other EMR. The box acts as a faraday cage. The only signal that can get into the box is the one that is passed down the TV aerial and is able to pass through the circuitry that selects the radio or TV channel. All other signals are excluded.

5a) $F = q \times E$

$F = 5.00 \times 10^{-9} \times 2.20 \times 10^{4}$
$\mathbf{F = 1.10 \times 10^{-4}\ N}$

$W = F \times s$
$W = 1.10 \times 10^{-4} \times 3.00 \times 10^{-3}$
$\mathbf{W = 3.30 \times 10^{-7}\ J}$

b) $W = q \times V$
$V = W / q$
$V = 3.30 \times 10^{-7} / 5.00 \times 10^{-9}$
$\mathbf{V = 66.0\ V}$

6. $V = E\ s$
$E = V / s$
$E = 12 / 120 \times 10^{-3}$
$\mathbf{E = 1.00 \times 10^{2}\ N\ C^{-1}}$ **or** $\mathbf{1.00 \times 10^{2}\ V\ m^{-1}}$ **(these are equivalent units)**

7a) $W(eV) = q\ V / e$
$W(eV) = e\ V / e$
$W(eV) = V$
$W(eV) = 5.00 \times 10^{3}\ eV$

b) $W = q\ V$
$W = 1.6 \times 10^{-19} \times 5.00 \times 10^{3}$
$W = 8.00 \times 10^{-16}\ J$

8a) $W(eV) = q\ V / e$
$W(eV) = 2e\ V / e$
$W(eV) = 2V$
$W(eV) = 2 \times 5.00 \times 10^{3}\ V$
$W(eV) = 1.00 \times 10^{4}\ eV$

8b) $W = q\,V$
$W = 2 \times 1.6 \times 10^{-19} \times 5.00 \times 10^{3}$
$W = 1.60 \times 10^{-15}$ J

9a) $V = E\,s$
$E = V / s$
$E = 1.50 \times 10^{4} / 2.70 \times 10^{-4}$
$E = 5.55 \times 10^{7}$ N C^{-1}

b) $W = q \times V$
$W = 1.6 \times 10^{-19} \times 1.50 \times 10^{4}$
$W = 2.4 \times 10^{-15}$ J

10a) The shirt has a static charge on it from the outer surface of the car. The dust particles became charged by induction and, hence, were opposite that of those on the shirt and so the dust was attracted.

b) $F = qE$
$F = 4 \times 10^{-6} \times 9$
$F = 3.60 \times 10^{-5}$ N

c) Assuming that the electric field is uniform:

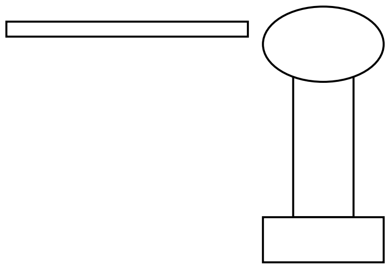
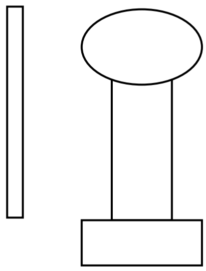
$W = F \times s$
$3.6 \times 10^{-7} = 3.6 \times 10^{-5}$ x s
s = 0.01 m

$W = q \times V$
$3.6 \times 10^{-7} = 4$ x $10^{-6} \times V$
$V = 9 \times 10^{-2}$ V

11a) An electrostatic precipitator uses a static charge to attract the dust and soot out of the smoke and gasses passing up a chimney or smoke stack. This leave gases that come out the end of the chimney clear of dust and soot.

b) A power company would install one to meet pollution regulations governing the type of waste that it can pollute the environment with.

12.

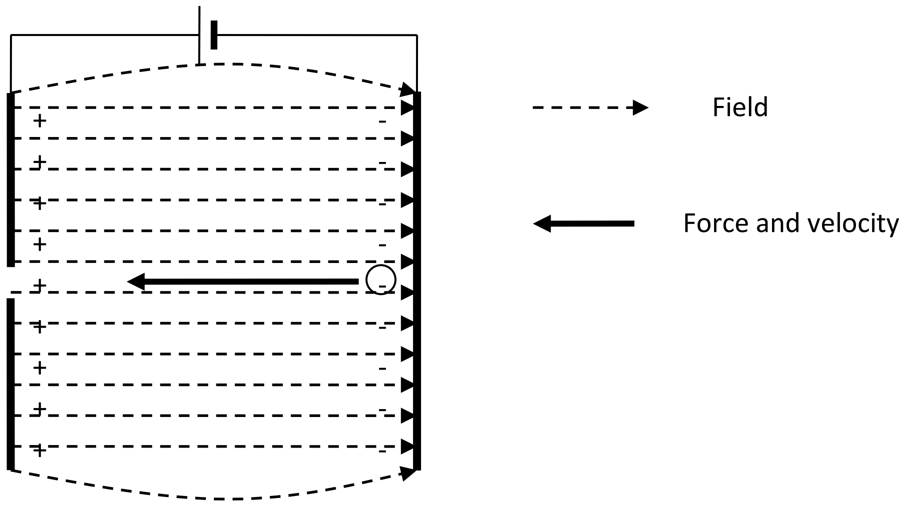
Light Produced	No Light produced
 The tube is in line with the electric field lines which accelerate charged particles (electrons) along the tube causing the light to work.	 The tube is parallel line with the electric field lines which accelerate charged particles (electrons) along the tube causing the light to work.

13. $qV = \frac{1}{2} m v^2$

$1.6 \times 10^{-19} \times 800 = 0.5 \times 1.67 \times 10^{-27} \times v^2$

$\mathbf{v = 3.92 \times 10^{5}\ m\ s^{-1}}$

14a)



b)

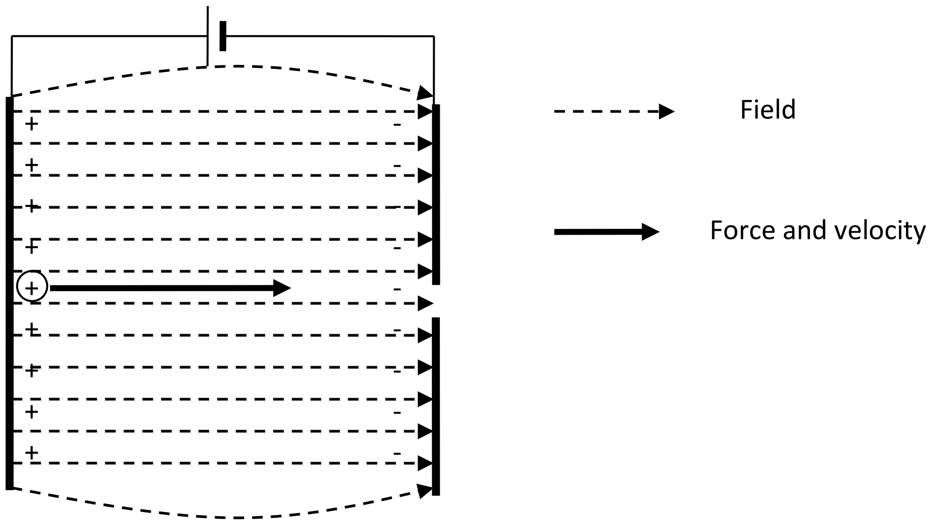
Method 1	Method 2
$F = qE$	$V = E \times s$
$F = 1.6 \times 10^{-19} \times 2.5 \times 10^{4}$	$V = 2.5 \times 10^{4} \times 0.1$
$\mathbf{F = 4.00 \times 10^{-15}\ N}$	$\mathbf{V = 2.5 \times 10^{3}\ V}$
$W = F \times s$	$W = q \times V$
$W = 4 \times 10^{-15} \times 10 \times 10^{-2}$	$W = 1.6 \times 10^{-19} \times 2.5 \times 10^{3}$
$\mathbf{W = 4 \times 10^{-16}\ J}$	$\mathbf{W = 4 \times 10^{-16}\ J}$

14c) $W = \frac{1}{2} m v^2$

$4 \times 10^{-16} = 0.5 \times 9.11 \times 10^{-31} \times v^2$

$\mathbf{v = 2.96 \times 10^{7}\ m\ s^{-1}}$ **This is less than the speed of light so probably ok.**

d)



e) Same kinetic energy or work as before because charge has not changed magnitude

$W = 4 \times 10^{-16}$ J

f) Final velocity of proton will be less than that of the electron because it has more mass. As m increases v^2 decreases.

$W = \frac{1}{2} m v^2$
$4 \times 10^{-16} = 0.5 \text{ x} 1.67 \times 10^{-27} \times v^2$
$v = 6.92 \times 10^5$ m s^{-1}

15a) $W = q \times V$

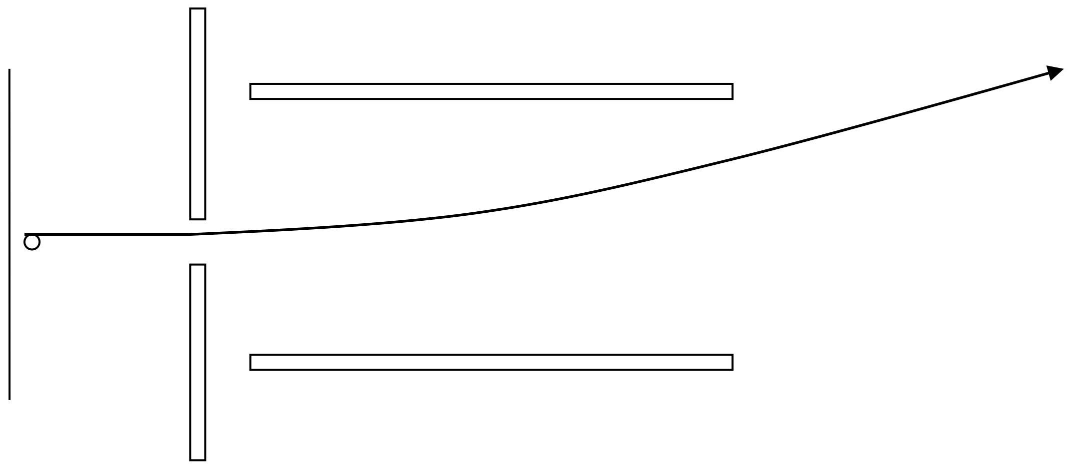
$W = 1.6 \times 10^{-19} \times 4000$
$W = 6.4 \times 10^{-16}$ J (4000 eV)

$W = \frac{1}{2} m v^2$
$6.4 \times 10^{-16} = \frac{1}{2} \times 9.11 \text{ x } 10^{-31} \times v^2$
$v = 3.75 \times 10^7$ m s^{-1}

b) $V = E\,s$
$E = V / s$
$E = 200 / 10.0 \times 10^{-2}$
$E = 2000$ N C^{-1} or 2000 V m^{-1}

$F = q \times E$
$F = 1.6 \times 10^{-19} \times 2000$
$F = 3.20 \times 10^{-16}$ N

15c)



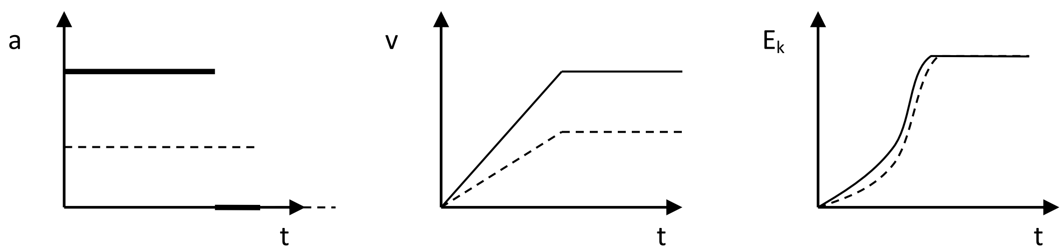
16a) If this were not done in a vacuum the accelerating electron would collide with the air gas molecules in the air and the electrons would be deflected or would be lost via ionisation with the molecules.

b) W = q × V
W = 1.6 × 10^{-19} × 2000
W = 3.20 × 10^{-16} J

W = F × s
F = W / s
F = 3.2 × 10^{-16} / 5 × 10^{-2}
F= 6.4 × 10^{-15} N

F = ma
6.4 × 10^{-15} = 9.11 × 10^{-31} x a
a = 7.03 × 10^{15} m s^{-2}

16c) and d)
i) ii) iii)



Answers to c

Answers to d

17. $q_1 = q_2 = 1.60 \times 10^{-19}$ C
∊ = 8.84 x 10^{-12} $C^2N^{-1}m^{-2}$
r = 5.30 x 10^{-11} m

$$F = \frac{1}{4pe}\frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi \times 8.84 \times 10^{-12}} \frac{(1.60 \times 10^{-19})^2}{(5.30 \times 10^{-11})^2}$$

$$F = 9.002 \times 10^{9} \times 9.114 \times 10^{-18}$$

$$F = 8.20 \times 10^{-8} N$$

18. $q_1 = 7.00 \times 10^{-9}$ C
$q_2 = 9.00 \times 10^{-9}$ C
$\varepsilon = 4.18 \times 10^{-11}\ C^2N^{-1}m^{-2}$
r = 25.0 cm = 0.250 m

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi \times 4.18 \times 10^{-11}} \frac{7.00 \times 10^{-9} \times 9.00 \times 10^{-9}}{0.250^2}$$

$$F = 1.904 \times 10^{9} \times 1.008 \times 10^{-15}$$

$$F = 1.92 \times 10^{-6} N$$

19. $q_1 = q_2 = 1.20 \times 10^{-6}$ C
ε = ?
r = 68.4 cm = 0.684 m
F = 1.86 N

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$$

$$1.86 = \frac{1}{4\pi \times \varepsilon} \frac{(1.20 \times 10^{-6})^2}{0.684^2}$$

$$1.86 = \frac{1}{4\pi \times \varepsilon} \times 3.078 \times 10^{-12}$$

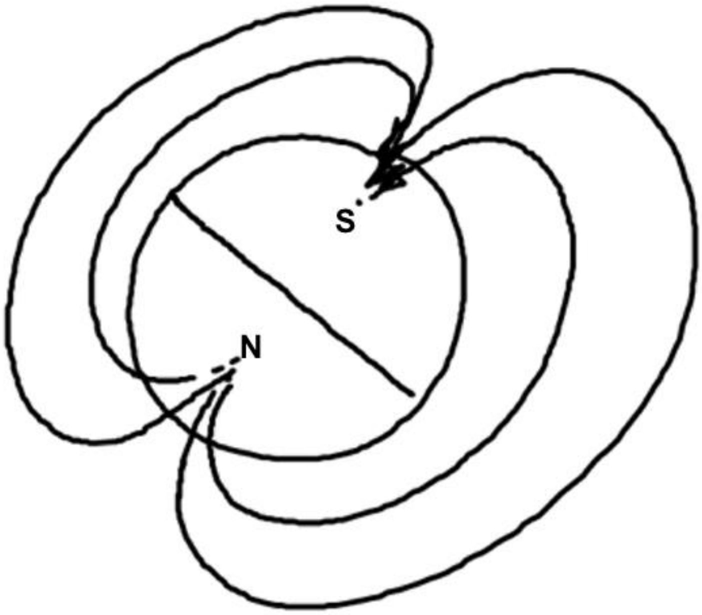
$$\varepsilon = \frac{1}{4\pi \times 1.86} \times 3.078 \times 10^{-12}$$

$$\varepsilon = 1.32 \times 10^{-13} C^2 N^{-1} m^{-2}$$

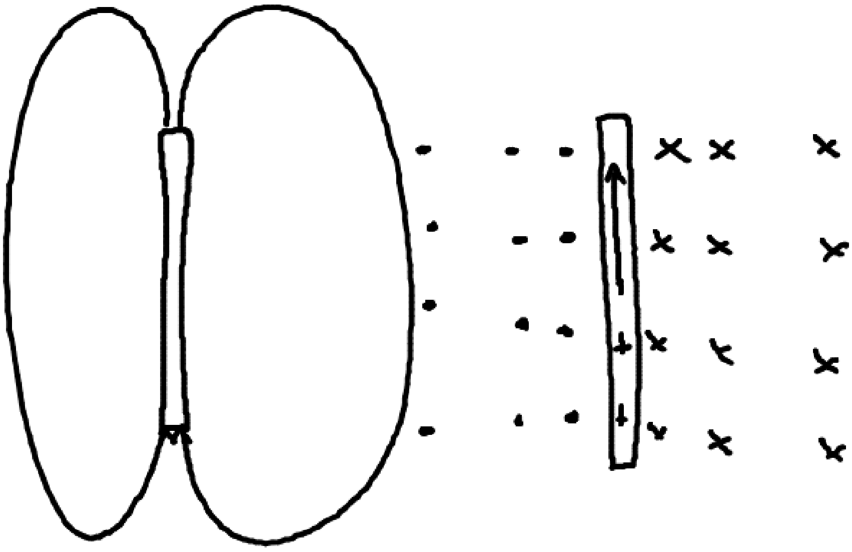
Problem Set 10: Charged particles in a magnetic field

1.a) The space or region around a magnet or moving electric charge within which the magnetic force operates.

b) Sketch magnetic field of earth. Note that magnetic north roughly corresponds with geographic south, magnetic south with geographic north

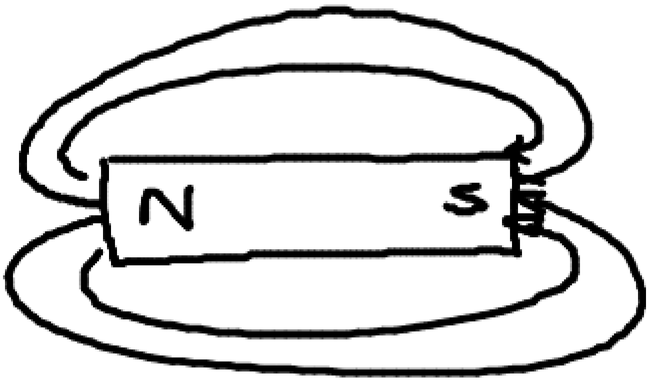


c) magnetised wire v's current carrying wire.

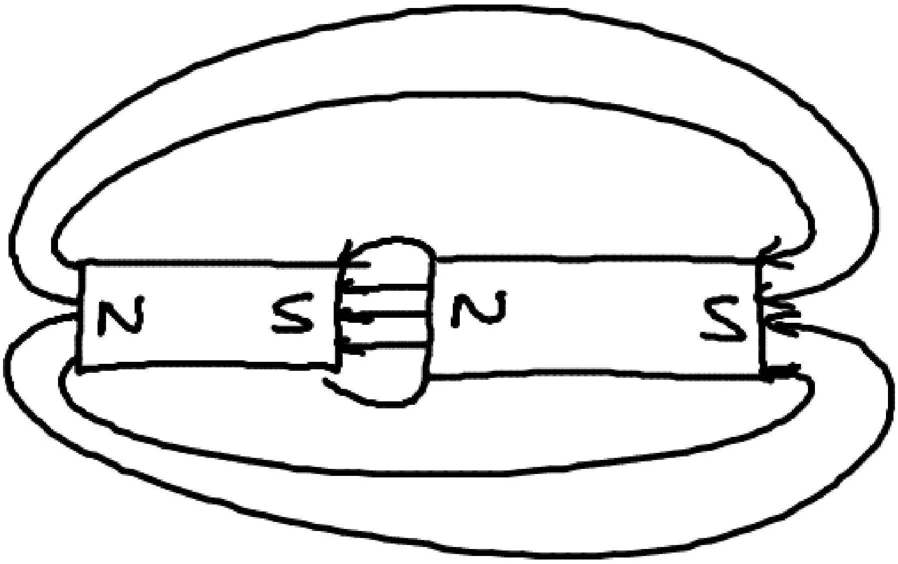


d) Coil the wire into a solenoid

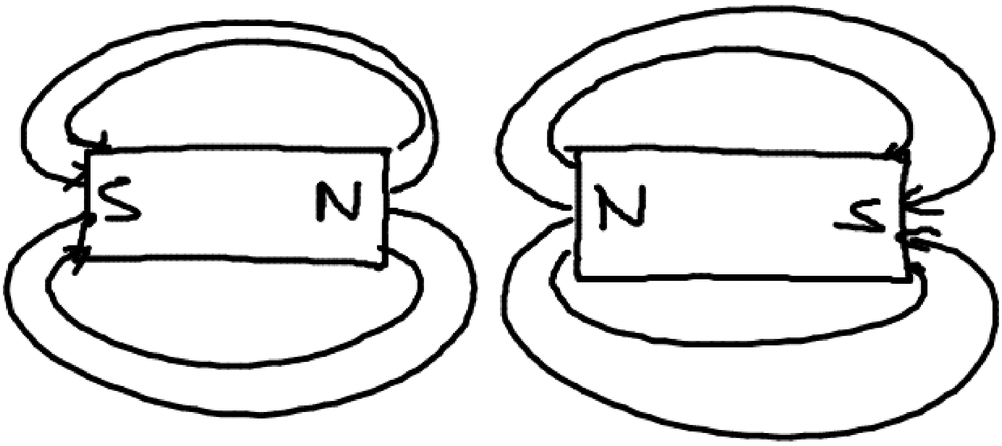
2.a) Bar magnet



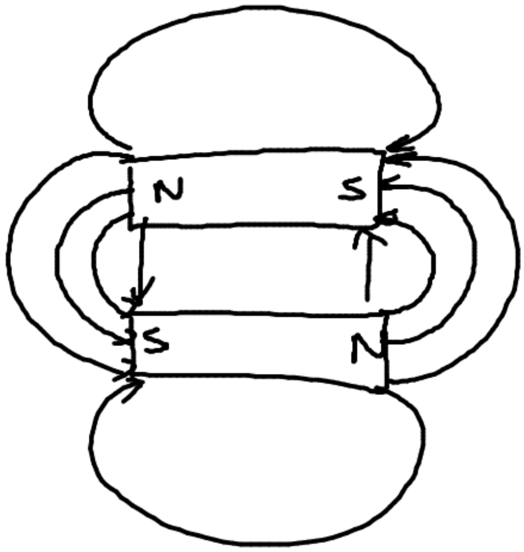
b) 2 bar magnets in line north to south



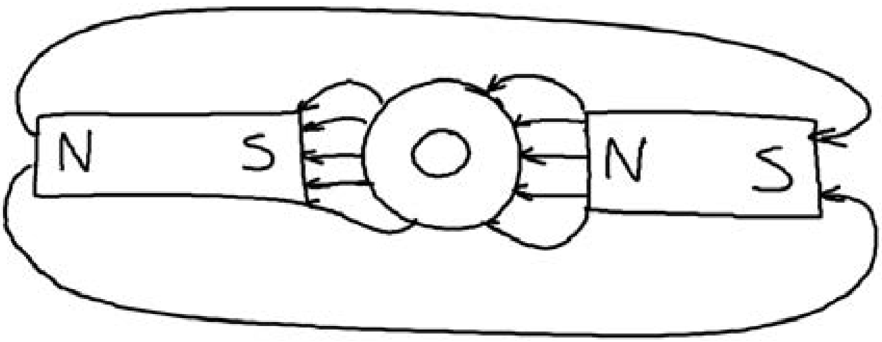
c) 2 bar magnets in line north to north



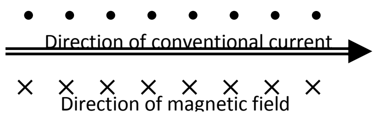
d) 2 bar magnets side by side

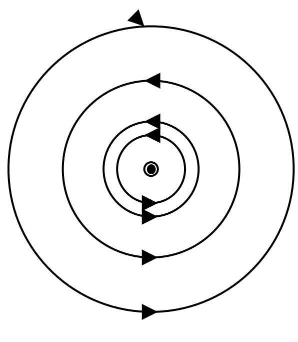


e) 2 bar magnets north to south with washer in between.

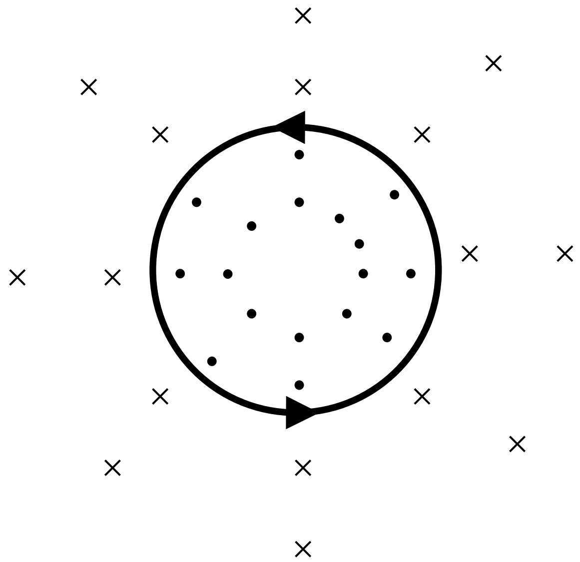


3.a) Current carrying conductor.





b) Current carrying coil of wire



4.a) No change in direction. Velocity is parallel with field.
b) Electron will change direction because of a force according to the right hand rule.

5. A is positive B is uncharged, C is negative

6.a) F = qvB
b) Spiral with a radius dependent upon the proton"s initial velocity $mv^2/r = qvB$ $r = mv / qB$
c) $v = 2\pi r / T$ $v= 2\pi rf$
d) Sub $v= 2\pi rf$ formula into $r = mv / qB$

$rqB = m(2\pi rf)$
$qB = m(2\pi f)$

$f = \frac{qB}{2\pi m}$

f is proportional to B
$f = \frac{qB}{2\pi m}$

f is proportional to v
$v= 2\pi rf$

7. Min radius = diameter/2 = 0.0990/2 = 0-0495 m
Max radius = diameter/2 + width of the slit = 0.0495+0.002 = 0.0515

r = 0.0495 m (minimum radius)	r = 0.0515 m (maximum radius
$r = mv / qB$ $m = rqB / v$ $m = 0.0495 \times 1.6 \text{ x } 10^{-19} \times 10.0 / 5.00 \times 10^6$ **$m = 1.58 \times 10^{-26}$ kg**	$r = mv / qB$ $m = rqB / v$ $m = 0.0515 \times 1.6 \times 10^{-19} \times 10.0 / 5.00 \times 10^6$ **$m = 1.65 \times 10^{-26}$ kg**

8.a) $r = mv / qB$
$r = 1.67 \times 10^{-27} \times 1 \times 10^4 / 1.6 \times 10^{-19} \times 2.50 \times 10^{-6}$
r = 41.75 m
r = 41.8 m (3 sig fig)

b) $v = 2\pi r / T$
$T = 2\pi r / v$
$T = 2\pi \times 41.75 / 1 \times 10^4$

$T = 2.62 \times 10^{-2}$ s

c) As B increases T decreases

$r = mv / qB$

but…

$v = 2\pi r / T$

substituting we get

$r = m\,2\pi r / qBT$

Re arranging for T we get

$T = m2\pi / qB$

Hence as B increases T decreases

d) As the v increases the radius of the circle increases according to $r = mv / qB$
If the radius of the circle increase the circumference increases by the formula $2\pi\,(mv / qB)$
Hence the time taken is constant. by the formula $\mathbf{v} = 2\pi\,(m\mathbf{v} / qB) / T$.

9.a) B_1 = into the page, B_2 = out of the page.

b) $F = qvB$
$F = 1.6 \times 10^{-19} \times 1.50 \text{ x } 10^{6} \times 0.1$
$F = 2.40 \times 10^{-14}$ N

c) The electrons are in the field for longer. The causes them to experience a force for longer and increases the amount of deflection they experience.

d) Make the field non uniform so that the field is stronger at the top and bottom edges of the rectangle. The direction of the field will reverse half way down the triangle.

10.a) $qvB = mv^2 / r$

$r = mv / qB$

$q / m = v / rB$

b) i) $q / m = v / rB$
$q / m = 2.20 \times 10^5 / 2.9 \times 10^{-2} \times 0.120$
$q / m = 6.32 \times 10^7$ C / kg (6.3 × 10^7 C / kg)

ii) $q / m = v / rB$
$q / m = 2.20 \times 10^5 / 3.8 \times 10^{-2} \times 0.120$
$q / m = 4.82 \times 10^7$ C / kg (4.8 × 10^7 C / kg)

Ratio = 6.52 / 4.82 = **1.35 : 1**

c) Yes. The radius are in a 1/3 : 1/4 ratio based on reciprocal of mass.

d) Solve for mass using $m = qrB / v$

Divided the mass found by the mass of a proton or neutron (1.67 × 10^{-27}) to discover the number of nucleons.

Oxygen - 16, Oxygen - 17 and Oxygen – 18 (in order from smallest radius to largest).

11. $v = 0.5c = 0.5(3.00 \times 10^8)$ m s^{-1} = 1.50×10^8 m s^{-1}
$r = 4$ cm $= 0.04$ m

$$r = \frac{mv}{qB}$$

$$B = \frac{mv}{qr}$$

$B = 37.5 \times 10^8$ m/q $= 3.75 \times 10^9$ m/q

where m = mass of the positively charged ion
and q = charge on the ion
The field strength can be found once the postively charged ion is identified.

Problem Set 11: Charged particles in combined electric and magnetic fields

1.a) Proton accelerates parallel with field. Proton may accelerate positively or negatively depending on the direction of the field and sign convention.

b) No force on charged particle moving parallel with field lines.

c) Proton accelerates parallel with field. Proton may accelerate positively or negatively depending on the direction of the field and sign convention. proton is deflected sideways relative to original line of motion.

d) Proton begins to move in a circle because it is experiencing a force as it cuts across magnetic field lines.

2. As seen from a top view looking down, electric field is west to east. The magnetic field will need to be North to South.

3.a) $qvB = qE$
$4.5 \times 10^6 \times 24.5 \times 10^{-3} = E$
$\mathbf{E = 1.10 \times 10^5\ N\ C^{-1}}$

b) Yes - the forces are independent of mass

4.a) The field has a turning effect on the electron.

b) The electron passes straight through un-deviated. There is no force on the electrons when travelling parallel to the magnetic field.

c) Electron decelerates linearly and bounces back in the same direction for which it came.

$ma = qE$
$a = qE / m$
$a = 1.6 \times 10^{-19} \times 1.00 \times 10^3 / 9.11 \times 10^{-31}$
$a = 1.76 \times 10^{14}\ m\ s^{-2}$ in the opposite direction to the field

5.a) $qV = ½\ m\ v^2$
$v^2 = 2 \times 1.6 \times 10^{-9} \times 10.0 \times 10^3 / 9.11 \times 10^{-31}$
$v^2 = 5.27 \times 10^{15}$
$\mathbf{v = 7.26 \times 10^7\ m/s}$

b) $r = mv / qB$
$r = 9.11 \times 10^{-31} \times \mathbf{7.26 \times 10^7 / 1.6 \times 10^{-19} \times 2.35}$
$r = 1.76 \times 10^{-4}$ m or 0.176 mm

c) $v = 2\pi r / T$
$T = 2\pi r / v$
$T = 2\pi \times 1.76 \times 10^{-4} / 7.26 \times 10^7$
$\mathbf{T = 1.52 \times 10^{-11}\ s}$

6.a) $r = mv / qB$

$v = rqB / m$

$v = 0.05 \times 1.6 \times 10^{-19} \times 1.50 / 1.67 \times 10^{-27}$

$\mathbf{v = 7.19 \times 10^{6}\ m/s}$

b) $v = 2\pi r / T$

$T = 2\pi r / v$

$\mathbf{T = 4.37 \times 10^{-8}\ s}$

$f = 1 / T$

$\mathbf{f = 2.29 \times 10^{7}\ Hz}$

c) $qV = ½\, m\, v^2$

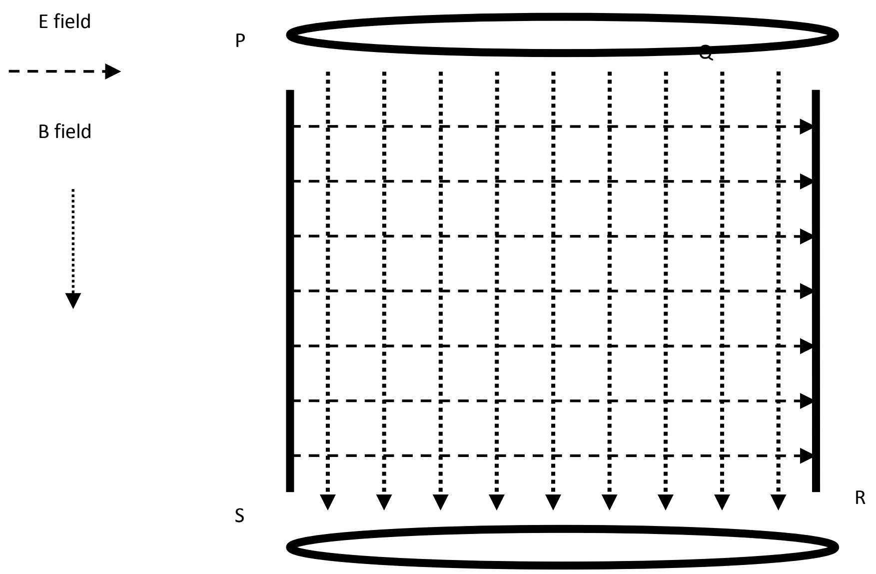
$1.6 \times 10^{-19} \times V = ½\ 1.67 \times 10^{-27} \times (\mathbf{7.19 \times 10^{6}})^{2}$

$\mathbf{V = 2.70 \times 10^{5}\ V}$ **or 270 kV**

7.a)

	E field	B field
Deflection direction (positive charge)	Deflected with field	If at right angles to field deflected at right angles to velocity and field according to the right hand rule.
Effect of velocity	Does not affect the magnitude (force) of deflection (F = qE) (no v term)	Does effect the force of deflection (F = qvB) (Contains v term)

b)



c) $F = q E$

$F = qvB$

$E = vB$

$E / B = v$

$1.00 \times 10^{4} / 0.100 = v$

$\mathbf{v = 1.00 \times 10^{5}\ m/s}$

8.a) $qV = ½\, m\, v^2$

$v^2 = 2\ q\ V\ /\ m$
$v^2 = 2 \times 1.6 \times 10^{-19} \times 20.0 \times 10^3\ /\ 1.67 \times 10^{-27}$
$v = 1.96 \times 10^6$ m/s

$F = qvB$
$F = ma$
$a = qvB\ /\ m$
$a = 1.6 \times 10^{-19} \times \mathbf{1.96 \times 10^6 \times 0.200\ /\ 1.67 \times 10^{-27}}$
$a = 3.76 \times 10^{13}$ m s^{-2} at right angles to the field

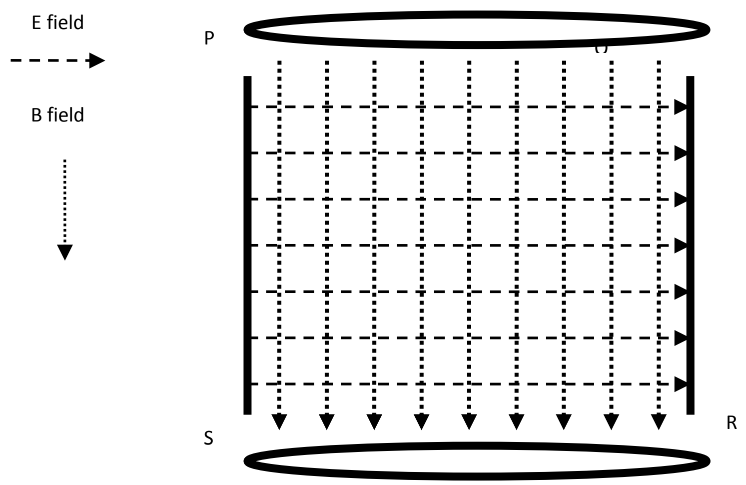
b) The force is a centripetal force, acting at right angles to the particles velocity toward the centre of the circular path.

c) $qE = qvB$
$E = vB$
$E = 1.96 \times 10^6 \times 0.200$
$E = 3.92 \times 10^5$ N C^{-1}

9.a) $r = mv\ /\ qB$
$r = 9.11 \times 10^{-31} \times 1.6 \times 10^4\ /\ 1.6 \times 10^{-19} \times 3.00 \times 10^{-2}$
$r = 3.04 \times 10^{-6}$ m

b) $E = vB$
$E = 1.6 \times 10^4 \times 3.00 \times 10^{-2}$
$E = 480$ N C^{-1}

c)



Electron is fired into the page.

d) Original electron travelling at 1.6×10^4 m/s passes through un-deviated: faster electron is bent more by the magnetic field and so in the above diagram, bents to the right.

10 Figures partly obscured in the diagram. Larger radius should read 0.40 m not 40 m

a) The radius increases because the velocity of the charged particle increases as it moves from one D to the next. As v increases r increases.

b) i)

$r = mv / qB$
$v = rqB / m$
$v = 0.20 \times 1.6 \times 10^{-19} \times 1.50 / 1.67 \times 10^{-27}$
$\mathbf{v = 2.87 \times 10^{7}\ m/s}$

ii)

$r = mv / qB$
$v = rqB / m$
$v = 0.40 \times 1.6 \times 10^{-19} \times 1.50 / 1.67 \times 10^{-27}$
$\mathbf{v = 5.75 \times 10^{7}\ m/s}$

Problem Set 12.1: Waves and Photons

1. The comment that light can be transmitted as an electromagnetic wave refers to the face that light can be modelled by coupled transverse waves, one a magnetic field and the other an electrical field, at right angles to each other, that move forward by creating each from the other at the speed of light.

2. a) The term coherent is used to describe light sources of the same frequency, and that have a constant phase difference (ie they are in phase.).

   b) No – because they are producing more than one frequency of EM radiation. Coherent light must be monochromatic, as even if the light sources produced the same range of frequencies, there is no reason why both sources will produce the same frequency at the same time.

3. According to the formula $E = hf$ the energy of a photon is reliant not on the velocity of the photon, but on its frequency. Different colours of light correspond to photons of different frequencies, and therefore, according to $E = hf$, different energies.

4. $E = hf$
   $E = (6.63 \times 10^{-34})(3.85 \times 10^{14})$
   $E = 2.55 \times 10^{-19}$ J

5. $E = hf$
   $1.00 \times 10^{-17} = (6.63 \times 10^{-34})f$
   $f = 1.58 \times 10^{16}$ Hz

   This corresponds to the Ultraviolet region of the EM spectrum

6. a) $E = hf$
   $E = (6.63 \times 10^{-34})(1.30 \times 10^{6})$
   $E = 8.62 \times 10^{-28}$ J

   b) $c = \lambda f$
   $\lambda = \frac{3.00\times10^{8}}{1.30\times10^{6}}$
   $\lambda = 2.31 \times 10^{2}$ m

   c) $E = \frac{1}{2}\, m\, v^2$
   $8.62 \times 10^{-28} = \frac{1}{2}(9.11 \times 10^{-31})v^2$
   $v = 4.35 \times 10^{1}$ ms^{-1}

7. a) $c = \lambda f$
   $\lambda = \frac{3.00\times10^{8}}{2650\times10^{6}}$
   $\lambda = 1.13 \times 10^{-1}$m

   b) $E = hf$
   $E = (6.63 \times 10^{-34})(2650 \times 10^{6})$
   $E = 1.76 \times 10^{-24}$ J

c) Energy after 2.5 mins:

$E_{tot} = P.t$

$E_{tot} = 1100 \times 2.5 \times 60$

$E_{tot} = 165000$ J

$n_{photon} = \frac{E_{tot}}{E_{photon}}$

$n_{photon} = \frac{165000}{1.76\times10^{-24}}$

$n_{photon} = 9.38 \times 10^{27}$ photons

8. a) UV

b) $E = hf$

$E = \frac{hc}{\lambda}$

$E = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{905\times10^{-9}}$

$E = 2.20 \times 10^{-19}$ J

c) $E_{pulse} = P.t$

$E = 34(150 \times 10^{-9})$

$E = 5.1 \times 10^{-6}$ J

$n_{photons} = \frac{E_{pulse}}{E_{photon}}$

$n_{photon} = \frac{5.1\times10^{-6}}{2.20\times10^{-19}}$

$n_{photon} = 2.32 \times 10^{13}$ photons

9. Antennas need to be mounted in an orientation which corresponds to the method of polarisation the waves they're receiving have been subject to. Horizontal antennas are placed to receive TV signals, as the TV signals are horizontally polarised themselves.

10. a) $c = \lambda f$

$\lambda = \frac{3.00\times10^{8}}{1.52\times10^{9}}$

$\lambda = 1.97 \times 10^{-1}$ m

b) $E = hf$

$E = (6.63 \times 10^{-34})(1.52 \times 10^{9})$

$E = 1.01 \times 10^{-24}$ J

c) $n_{photons\ in\ 1\ second} = \frac{E_{energy\ released\ in\ 1\ second}}{E_{photon}}$

$n_{photons\ in\ 1\ second} = \frac{5.00}{1.01\times10^{-24}}$

$n_{photons\ in\ 1\ second} = 4.95 \times 10^{24}$ photons in 1 second

d) Assuming time elapsed starts with the first pulse:

250 pulses in 1 second.

$$E_{single\ pulse} = \frac{E_{1\ second}}{E\ pulses\ in\ 1\ second}$$
$$E_{single\ pulse} = \frac{5.00}{250}$$
$$E_{single\ pulse} = 2.00 \times 10^{-2}\text{J}$$

11. $c = \lambda f$

$$f = \frac{3.00 \times 10^{8}}{435 \times 10^{-9}}$$
$$f = 6.90 \times 10^{14}\text{Hz}$$

$$E = hf$$
$$E = (6.63 \times 10^{-34})(6.90 \times 10^{14})$$
$$E = 4.57 \times 10^{-19}\text{J}$$

$$P = E_{photon} \cdot n_{photons\ in\ 1\ second}$$
$$P = 4.57 \times 10^{-19}. 3.25 \times 10^{18}$$
$$P = 1.49\text{W}$$

$$W = P.t$$
$$W = 1.49 \times 1.2 \times 10^{-3}$$
$$W = 1.78 \times 10^{-3}\text{J}$$

12. a) $E = hf$

$$E = \frac{hc}{\lambda}$$
$$E = \frac{(6.63\times10^{-34})(3.00\times10^{8})}{694\times10^{-9}}$$
$$E = 2.87 \times 10^{-19}\text{ J}$$

b) $I = \frac{P}{A}$

$$I = \frac{1.00}{10\times10^{-6}}$$
$$I = 100000\text{ W.m}^{-2}$$

c) Laser is 100x brighter.

13. $E = hf$

$$E = \frac{hc}{\lambda}$$
$$E = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{6.00 \times 10^{-7}}$$
$$E = 3.32 \times 10^{-19}\text{ J}$$

$$n_{photons\ in\ 1\ second} = \frac{E_{energy\ released\ in\ 1\ second}}{E_{photon}}$$
$$n_{photons\ in\ 1\ second} = \frac{1.70 \times 10^{-8}}{3.32 \times 10^{-19}}$$
$$n_{photons\ in\ 1\ second} = 5.13 \times 10^{10}\text{ photons in 1 second}$$

14. a) $c = \lambda f$

$\lambda = \frac{3.00\times10^{8}}{720\times10^{3}}$

$\lambda = 4.17 \times 10^{2}$m

b) Assuming the full power is used for the day:

$E = P.t$

$E = 50 \times 10^{3} \times 60 \times 60 \times 24$

$E = 4.32 \times 10^{9}$J

15. a) $E = hf$

$f = \frac{E}{h}$

$f = \frac{2.00\times10^{-24}}{6.63\times10^{-34}}$

$f = 3.02 \times 10^{9}$ Hz

b) $c = \lambda f$

$\lambda = \frac{3.00\times10^{8}}{3.02\times10^{9}}$

$\lambda = 9.95 \times 10^{-2}$m

c)

	Compared to VL
E	Smaller
f	Lower
λ	Greater

16. a) Green light has a higher frequency than the IR beam, which means it is less likely to interact with the molecules on the way through the water.

b) High intensity means the beam consists of many photons. It needs a high intensity because photons may scatter due to collisions with molecules in the water. The more photons, the more photons that will successfully be reflected back.

c) Wavelength = $.75 \times 532 = 399$ nm

d) Frequency doesn't change between different media

$f = \frac{c}{\lambda}$

$f = \frac{3\times10^{8}}{532\times10^{-9}}$

$f = 5.64 \times 10^{14}$ Hz

e) Speed of light in water is roughly 75% of the speed of light in air.

$speed = \frac{distance}{time\ from\ bottom\ of\ ocean\ to\ top}$

$0.75 \times 3 \times 10^{8} = \frac{distance}{\frac{1}{2}(3.8\times10^{-6})}$

$distance = 427.5$ m

17. a) Incandescent globes only emit a small proportion of energy supplied as visible light. A large proportion of the energy supplied is radiated as heat; EM radiation in the Infrared range.

b) Blackbody curve should have its peak in the UV part of the spectrum.

c) $c = \lambda f$

$\lambda = \frac{3.00 \times 10^8}{5.28 \times 10^{16}}$

$\lambda = 5.68 \times 10^{-9}$ m

d) $E = hf$

$E = (6.63 \times 10^{-34})(5.28 \times 10^{16})$

$E = 3.50 \times 10^{-17}$J

$E = 2.19 \times 10^2$ eV

e) $n_{photons\ in\ 1\ second} = \frac{E_{energy\ released\ in\ 1\ second}}{E_{photon}}$

$n_{photons\ in\ 1\ second} = \frac{75.0}{3.50 \times 10^{-17}}$

$n_{photons\ in\ 1\ second} = 2.14 \times 10^{18}$ photons in 1 second

f) It would be hard to determine the number of photons from experimental measure, as the photons are not travelling in a beam towards a detector like in a laser, but radially. This is impractical to detect due to the fact they can travel in any direction.

Problem Set 12.2: The photoelectric effect

1. The photoelectric effect supported the particle model of light in two significant ways. Firstly, the fact there was a threshold frequency which determined whether or not a photoelectron current would flow supported the idea that each quantum of light contains a certain amount of energy; dependant on the frequency of that light. Secondly, the effect of increasing the intensity of the light on the photoelectron current when the light source is above the threshold frequency also supports the idea that the number of photons is related to the intensity.

2. $E = \frac{hc}{\lambda}$
   $h = \frac{E\lambda}{c}$
   $h = \frac{4.00 \times 10^{-7} \times 1.40 \times 10^{-19}}{3 \times 10^{8}}$
   $h = 1.87 \times 10^{-34}\text{J.s}$

   $h = \frac{3.00 \times 10^{-7} \times 3.06 \times 10^{-19}}{3 \times 10^{8}}$
   $h = 3.06 \times 10^{-34}\text{J.s}$

3. $E_K = hf - W$
   $E_K = 6.63 \times 10^{-34} \times 6.7 \times 10^{14} - 2.14 \times 1.60 \times 10^{-19}$
   $E_K = 1.02 \times 10^{-19}$ J

4. a) The minimum energy required of a photon incident on the metal surface to liberate an electron.

   b) $E = \frac{hc}{\lambda}$
   $\lambda = \frac{hc}{E}$
   $\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{4.08\times1.60\times10^{-19}}$
   $\lambda = 3.04 \times 10^{-7}\text{m}$

   c) $E_K = hf - W$
   $E_K = 6.63 \times 10^{-34} \times 2.3 \times 10^{15} - 4.08 \times 1.60 \times 10^{-19}$
   $E_K = 9.32 \times 10^{-19}$ J

5. a) No effect

   b) No effect

   c) No effect

   d) Different metals have different work functions, which mean there are differences in the minimum frequency of light that will cause photoelectrons to be emitted from the metal (the threshold frequency).

   e) According to $= \frac{c}{f}$, the wavelength and frequency are inversely proportional. Increasing the wavelength will cause a decrease in the frequency of the light. If this drops below the threshold frequency, no photoelectrons will be emitted.

f) If the frequency is above the threshold frequency, a higher intensity will increase the photoelectron current. If the frequency is below the threshold frequency, no current will flow, and changing the intensity will not change this.

g) If the surface of the metal is covered (and the material is opaque) light will not be able to strike the surface of the metal. If this is the case, the photoelectric effect will not be seen.

6. a) $E_{photon} = \frac{hc}{\lambda}$

$E_{photon} = \frac{6.63\times10^{-34}\times3\times10^{8}}{325\times10^{-9}}$

$E_{photon} = 6.12 \times 10^{-19}\text{J}$

$E_{photon} = 3.83 \text{ eV}$

$E_K = E_{photon} - W$

$E_K = 3.83 - 5.01$

No solution as the incident energy is below the work function.

b) No effect. The stopping potential is related to the maximum kinetic energy of the electrons that are liberated. Doubling the intensity changes the amount of photoelectrons which are liberated, but not the energy of those electrons (this is related to increasing the frequency of the light). Therefore, there is no effect on the stopping potential.

7. a) At the threshold frequency, electrons are free but have no kinetic energy.

$E_K = hf - W$

$0 = hf - W$

$W = 1.25 \times 10^{14} \times 6.63 \times 10^{-34}$

$W = 8.29 \times 10^{-20}\text{J}$

$W = 5.18 \times 10^{-1}\text{eV}$

b) $\lambda = \frac{c}{f}$

$\lambda = \frac{3.00 \times 10^{8}}{6.95 \times 10^{14}}$

$\lambda = 4.31 \times 10^{-7}\text{m}$

c) $E = hf$

$E = 6.63 \times 10^{-34} \times 6.95 \times 10^{14}$

$E = 4.61 \times 10^{-19}\text{J}$

$E = 2.88 \text{ eV}$

d) $E_K = hf - W$

$E_K = 2.88 - 5.18 \times 10^{-1}$

$E_K = 2.36 \text{ eV}$

e) $E_K = 2.36\text{eV} = 3.78 \times 10^{-19}\text{J}$

$E_K = \frac{1}{2}mv^2$

$3.78 \times 10^{-19} = \frac{1}{2}(9.11 \times 10^{-31})v^2$

$v = 9.11 \times 10^5 \text{m.s}^{-1}$

8. a) $E_{photon} = \frac{hc}{\lambda}$

$E_{photon} = \frac{6.63\times10^{-34}\times3\times10^8}{287\times10^{-9}}$

$E_{photon} = 6.93 \times 10^{-19}\text{J}$

$E_{photon} = 4.33 \text{ eV}$

$E_{K\ MAX} = 3.68 \text{ eV}$

$E_K = E_{photon} - W$

$3.68 = 4.33 - W$

$W = 6.51 \times 10^{-1}\text{eV}$

b) $E_{K\ MAX} = 3.68 \text{ eV} = 5.89 \times 10^{-19}\text{J}$

$E_{K\ MAX} = \frac{1}{2}mv^2$

$5.89 \times 10^{-19} = \frac{1}{2}(9.11 \times 10^{-31})v^2$

$v = 1.14 \times 10^6 \text{m.s}^{-1}$

9. a) In order for photoelectrons to be emitted from the surface the photons must have a minimum energy to dislodge them. As E=hf, this corresponds to photons of a minimum frequency. This is different for each material, however. 465nm light must correspond to a frequency which is higher than the threshold frequency in sodium, but not in platinum.

b) The current will increase in sodium as a higher intensity will mean more photons are produced, meaning more electrons will be liberated. As the light is below the threshold frequency for platinum, platinum's current will remain at zero.

c) $\lambda \,\alpha\, \frac{1}{f}$: increasing wavelength greatly greatly deceases frequency. Sodium will most likely stop producing electrons as a great decrease to frequency would put it below the threshold frequency, and platinum's current will remain at zero.

d) $\lambda \,\alpha\, \frac{1}{f}$: decreasing wavelength greatly increases frequency. Sodium will continue producing electrons at the same rate, and platinum is likely to produce a photoelectron current as a great increase to its frequency will most likely bring it above the threshold frequency.

e) Current will decrease in sodium, as a small reverse voltage will stop electrons with lower kinetic energy from moving.

10. a) $f = \frac{c}{\lambda}$

$f = \frac{3.00\times10^{8}}{3.55\times10^{-7}}$

$f = 8.45 \times 10^{14}$ Hz

b) $E = hf$

$E = 6.63 \times 10^{-34} \times 8.45 \times 10^{14}$

$E = 5.60 \times 10^{-19}$J

c) $E_K = E_{photon} - W$

$E_K = 5.60 \times 10^{-19} - 2.64 \times 10^{-19}$

$E_K = 2.96 \times 10^{-19}$J

d) $W = \frac{hc}{\lambda}$

$2.64 \times 10^{-19} = \frac{6.63\times10^{-34}\times3.00\times10^{8}}{\lambda}$

$\lambda = 7.53 \times 10^{-7}$ m

11. There is no energy change, but there will be a decrease in the number of photoelectrons (a decrease in the current). The light source does not change in frequency so the energy of the photons is constant throughout the entire process. The smoke, however, would cause photon to deflect from their path towards the photocell. This means less photons would interact with the electrons on the surface of the photocell, causing less photoelectrons to be emitted. This means the current would decrease.

12. bi) $f_0 = 9.00 \times 10^{14}$ Hz (x-intercept of graph)

ii) $h = 6.36 \times 10^{-34}$J.s (gradient of graph)

iii) $W = 5.71 \times 10^{-19}$J (y-intercept of graph)

Problem Set 13: Quantum Theory

1. a) These dark lines are absorption lines. When light travels through the elements and compounds in stars, photons of certain wavelengths are absorbed. The light from the star when viewed on Earth, therefore, is missing certain wavelengths. This causes the dark lines in the spectrum.

   b) Every absorption spectral line is also present in the emission spectra lines for a given element. By looking at the emission spectra for different elements in the lab we can account for all the lines, which are present in the absorption spectra of a star, and hence deduce which elements are in the star.

   c) The substance must be at a high temperature.

2. a) When white light is shone through the solution the molecules absorb certain frequencies of light. As white light contains all colours of light, the green colour is what we perceive to be the combination of all colours in white light without the colours which were absorbed.

   b) Absorption spectra.

3. a) $\Delta E = \frac{hc}{\lambda}$

   $\Delta E = \frac{6.63\times10^{-34}\times3\times10^{8}}{557.7\times10^{-9}}$

   $\Delta E = 3.57 \times 10^{-19}\text{J}$

   $\Delta E = 2.23 \text{ eV}$

   b)

4. **(i) E2 to E1**

   (a) $\Delta E = 10.2\ eV$

   $\Delta E = 1.63 \times 10^{-18}\text{J}$

   $\Delta E = hf$

   $f = \frac{1.63\times10^{-18}}{6.63\times10^{-34}}$

   $f = 2.46 \times 10^{15}\text{Hz}$

   (b) Ultraviolet

   (ii) E4 to E2

   (a) $\Delta E = 2.55eV$

   $\Delta E = 4.08 \times 10^{-19}\text{J}$

   $\Delta E = hf$

   $f = \frac{4.08\times10^{-19}}{6.63\times10^{-34}}$

   $f = 6.15 \times 10^{14}\text{Hz}$

   (b) Visible

   (iii) E3 to E2

   (a) $\Delta E = 1.89eV$

   $\Delta E = 3.02 \times 10^{-19}\text{J}$

   $\Delta E = hf$

   $f = \frac{3.02\times10^{-19}}{6.63\times10^{-34}}$

   $f = 4.56 \times 10^{14}\text{Hz}$

   (b) Visible

   (iv) E5 to E3

   (a) $\Delta E = 0.97eV$

   $\Delta E = 1.55 \times 10^{-19}\text{J}$

   $\Delta E = hf$

   $f = \frac{1.55\times10^{-19}}{6.63\times10^{-34}}$

   $f = 2.34 \times 10^{14}\text{Hz}$

   (b) Infrared

(c) $\Delta E = \frac{hc}{\lambda}$

$$\Delta E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{434 \times 10^{-9}}$$

$\Delta E = 4.58 \times 10^{-19}\text{J}$

$\Delta E = 2.86 \text{ eV}$

$E_5 \rightarrow E_2$ gives this difference.

5. $\Delta E = hf$

$\Delta E = 6.63 \times 10^{-34} \times 4.00 \times 10^{14}$

$\Delta E = 2.65 \times 10^{-19}\text{J}$

$\Delta E = 1.66 \text{ eV}$

$\text{E}_{higher} = 1.66 + E_{lower}$

$\text{E}_{higher} = 4.01\text{eV}$

6. Absorption spectra results when an electron in the ground state gains energy and transitions to a higher energy level. Emission spectra is the result of electrons moving from a higher energy level to a lower one; including all transitions to the ground state. Therefore any absorption spectra line will be present in the emission spectra.

7. a) 2.80 eV (no collisions), 1.42 eV (hits ground state goes to 1st), 0.5 eV (hits ground state goes to 2nd)

b) 1.38 eV (E2→E1), 2.30 eV (E3→E1), 0.92 eV (E3→E2)

c) Energies greater than or equal to 13.87 eV

d) The beam would be missing photons of energy 1.38 eV, 2.30 eV, and any photons of energy between 13.87 eV and 14.5 eV (inclusive).

8. (a) 2.06 eV

(b) $E = \frac{hc}{\lambda}$

$2.06 \times 1.60 \times 10^{-19} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$

$\lambda = 6.03 \times 10^{-7}\text{m}$

(c) $\frac{E\lambda}{c} = h$

$h = \frac{2.06 \times 1.60 \times 10^{-19} \times 600 \times 10^{-9}}{3 \times 10^{8}}$

$h = 6.59 \times 10^{-34} J.s$

9. $\Delta E = \frac{hc}{\lambda}$

$\Delta E = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{656.3 \times 10^{-9}}$

$\Delta E = 3.03 \times 10^{-19}\text{J}$

$\Delta E = 1.89 \text{ eV}$

10. $\Delta E = \frac{hc}{\lambda}$

$\Delta E = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5.84 \times 10^{-8}}$

$\Delta E = 3.41 \times 10^{-18}\text{J}$

$\Delta E = 2.13 \times 10^{1} \text{ eV}$

11. $E = \frac{hc}{\lambda}$

$E = \frac{6.63\times10^{-34}\times3\times10^{8}}{\lambda} \text{ J}$

$E = \frac{1.99\times10^{-25}}{\lambda} \text{ J}$

$E = \frac{1.99\times10^{-25}}{\lambda(1.6\times10^{-19})} \text{ eV}$

$E = \frac{1.24\times10^{-6}}{\lambda} \text{ eV}$

12. (a) $E_1 = \frac{hc}{\lambda}$

$E_1 = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.042\times10^{-7}}$

$E_1 = 1.91 \times 10^{-18} \text{ J} = 11.9 \text{ eV}$

$E_2 = \frac{hc}{\lambda}(1.6 * 10$

$E_2 = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.235\times10^{-7}}$

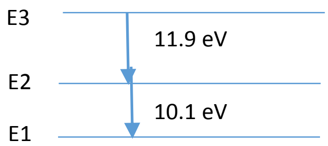
$E_2 = 1.61 \times 10^{-18} \text{ J} = 10.1 \text{ eV}$

$\lambda = \frac{hc}{E_1+E_2}$

$\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{3.52\times10^{-18}}$

$\lambda = 5.65 \times 10^{-8}\text{m}$

(b)



13. (a) 21.2 eV = 3.39 x 10^{-18} J photons.

$E = hf$
$3.339 \times 10^{-18} = 6.63 \times 10^{-34} f$
$f = 5.12 \times 10^{15}\text{Hz}$

UV

b) There would still be the same emission spectra, as electrons can give a portion of their kinetic energy after a collision. We should expect to see ionised helium, however.

c) The transmitted beam would be missing photons of energy 21.2eV, and anything equal to or above 24.6 eV. There is no change to the maximum photon which can be emitted, as there is still only one transition downwards, from the 21.2 eV energy level to the ground state.

Problem Set 14.1: Special relativity explanation questions

1. (a) An inertial reference frame is a region of space which has no net force acting on it – it is not accelerating as would be detected by an accelerometer. In inertial frames velocity is constant
(b) Not necessarily - they are both inertial frames but not the same frame. Measurements in one inertial frame can be converted to measurements in another by a simple transformation.

2. The laws of physics are the same in all inertial reference frames.
The speed of light in a vacuum is absolute and has the same value in all inertial reference frames.

3. "Proper" time is the time between two simultaneous events as seen and measured by an observer who is stationary relative to the object on which measurements are being made

4. Light is subject to Special Relativity, which says that anything with zero rest mass must travel through space-time at the speed of light. Light has zero rest mass and therefore, according to Special Relativity, must always travel through space-time at the speed of light. A photon has no experience of the passage of time so is everywhere at once.

5. The equation $E = mc^2$ applies only to particles with rest mass. A photon has energy and momentum but no rest mass and must travel at the speed of light. It has an energy proportional to its frequency but no minimum energy and if it stops moving it ceases to exist.

6. Having different velocities means they must be in different frames of reference. This includes having the same speed but moving in different directions or having different speeds in the same direction. In both of these cases there will be an acceleration because a change in velocity exists. According to special relativity if a frame is non-inertial it must be undergoing an acceleration and therefore not have a constant velocity. Two non-inertial frames will also be accelerating with respect to each other, unless they are accelerating in exactly the same way and in the same direction.

7. Velocities don't add up like they do in Newtonian mechanics. The relativity of space and time extends to velocity. The two velocities can be added together and the relative velocity for this data will be (0.70+0.70)c/(1+0.70 x 0.70) = 1.4c/1.49 = 0.94c.

8. The Lorentz contraction is not significant for an aircraft flying at a speed well below the velocity of light so there are no design issues to consider.
It is not necessary to worry about Lorentz contraction in any ship design, regardless of speed, because such contraction is relative and measured from a frame external to that of the ship. In the ship's own frame it has its proper length.

9. Density is mass per unit volume. Relativistic speed will result in a relativistic mass increase and a relativistic length decrease, therefore a relativistic increase in density.

10. When viewed in the reference frame of the starship the distance between the Earth and Alpha Centauri is seen as being length contracted and the clock as running normally.
To the Earth-based observer the spaceship clock is seen as being time dilated by the same factor but the distance is unchanged.
Observers in both locations agree the journey time is the same relative to their own frames of reference but their reasons are quite different –one being due to time dilation and the second to length contraction.

11. The relativistic mass of an object clearly increases as velocity increases. The length also decrease as at relativistic speeds. If length contracts then volume must decrease and as density = mass/volume, an increase in mass and a decrease in volume must result in an increase in density.

12. a) Length and time contract making $c = \Delta x/\Delta t$ constant

b) An observer outside would see that the spaceship would have changed its shape, being shorter in the direction of travel. However, in the spaceship's frame, everything has its proper length.

13. To a photon of light distance and time do not exist – so light is in effect subject to the effects mentioned

14. Light within a prism or some other medium appears to travel slower – this is due to the interaction of the electro-magnetic wave with electric and magnetic fields within the material.

15. $L = L_0 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2.2 \times \frac{1}{\sqrt{1-0.49}} = 3.1km$

16. According to you (in the spaceship), your clock runs exactly the same as it did when you were at rest on Earth, all objects in your ship appear the same to you as they did before, and the speed of light is still c. There is nothing you can do to find out if you are actually moving.

17. The speed of light is the same in all reference frames, independent of the speed of the source or the observer. Therefore, the light still travels at the speed c, and what you see in the mirror will be exactly the same as what you would see if you were at rest.

18. The scientist will observe the light beam reaching him at speed = c. Because of the principle of the constancy of the velocity of light, each observer will measure the light beam from the headlight as traveling at the same speed. This may be contrary to what you expected as you might have thought that the observer in oncoming spaceship would have measured the beam moving at (the speed of light) + (2 x the speed of the spaceship). Nevertheless, this is not what is observed in practice. What is actually occurs in the real world is that no one ever measures light moving faster or slower than c.

Problem Set 14.2: Special Relativity Calculations

1. $t = 2t_0$

$t = \gamma t_0$

$\therefore \gamma = 2$

$\therefore \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}$

$1-\frac{v^2}{c^2} = \frac{1}{4}$

$\frac{v^2}{c^2} = \frac{3}{4} = 0.75$

$v^2 = 0.75\ (3 \times 10^8)^2$

$v = \sqrt{0.75(3\times10^8)^2}$

$= 2.60 \times 10^8\ ms^{-1}$

2. $\ell = 0.7\ \ell_0$

$\ell = \frac{\ell_0}{\gamma}$

$\therefore \gamma = \frac{1}{0.7}$

$\therefore \sqrt{1-\frac{v^2}{c^2}} = 0.7$

$1-\frac{v^2}{c^2} = (0.7)^2$

$1-\frac{v^2}{c^2} = 0.49$

$\frac{v^2}{c^2} = 0.51$

$v^2 = 0.51\ (3 \times 10^8)^2$

$v = \sqrt{0.51\ (3\times10^8)^2}$

$v = 2.14 \times 10^8\ ms^{-1}$

3. $m_o = 2.5$ t

$V = 0.92c$

$p = \gamma mv$

$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \frac{2.5\times10^3 \times 0.92(3\times10^8)}{\sqrt{1-0.9^2}}$

$= \frac{6.75 \times 10^{11}}{\sqrt{0.19}}$

$= 1.55 \times 10^{12}\ kgms^{-1}$

4. $u' = 0.8c$

$u = \frac{v-u'}{1+\frac{vu'}{c^2}}$

$= \frac{(0.8+0.8)c}{1+0.64}$

$= \frac{1.6c}{1.64}$

$= 0.975c$

5. $t=1s$

$v=0.92c$

$t=\gamma\ t0$

$t_0 = \frac{t}{\gamma}$

$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \frac{1}{\sqrt{1-0.92^2}}$

$= 2.55$ s

6. $\ell_0 = 200$ m

$\ell = 160$ m

$\ell = \frac{\ell_0}{\gamma}$

$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}}$

$(\frac{\ell}{\ell_0})^2 = 1-\frac{v^2}{c^2}$

$(\frac{160^2}{200^2}) = 1-\frac{v^2}{c^2}$

$\frac{v^2}{c^2} = 1-0.8^2$

$v^2 = 0.36c^2$

v=0.6c

7a. v=0.995c

$\ell_0 = 250$ light years

$\ell = \frac{\ell_0}{\gamma}$

$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}}$

$= 250\sqrt{1-0.995^2}$

= 24.97 yr

$t = \frac{\ell}{v}$

$= \frac{24.97}{0.995}$ yr

= 25.1 yr

7b v=0.995v

ℓ =250 light year

$t = \frac{\ell}{v}$

8. v=0.994c

$\ell_0 = 250$ light years

t=3.00 x 10^{-8} s

$\ell = \upsilon_0 t$

=0.994 x 3.00 x 10^8 x 3.00 x 10^{-8}

=8.946 m

$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}}$

$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}}$

$8.946 = \ell_0\sqrt{1-0.992^2}$

$\ell_0 = \frac{8.946}{0.1094}$

=81.8 m

∴ hanger length 81.8 m

9. $t_{\frac{1}{2}}(\mu) = 26\ \mu s$

$\upsilon_0 = 0.95c$

$t = t_0\sqrt{1-\frac{v^2}{c^2}}$

$= \frac{26x10^{-6}}{\sqrt{1-0.95^2}}$

= 8.33 x 10^{-5} s

10. $\upsilon_E = 0.70c$

$\upsilon_D = 0.85c$

a. Earth observer would see them as 0.7c + 0.85c

b. Discovery would see Earth as moving away at 0.85c

c. Wrong because the speed of light is absolute.

$$u' = \frac{u+v}{1-\frac{uv}{c^2}}$$

$$= \frac{(0.85+0.70)c}{1+0.85x0.7}$$

$$== \frac{1.55}{1.615}c$$

=0.960c

11. $\ell = 1.2 \text{ x } 10^5$ light year

$\upsilon_0 = 0.98c$

$$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}}$$

$= 1.2 \text{ x } 10^5\sqrt{1\text{-}0.98^2}$

= 2.39 x 10^4 light years

$$t = \frac{\ell}{v}$$

$$= \frac{1.2 \text{ x } 10^5}{0.98}$$

=1.22 x 10^5 Yr

12. v=0.95c

$t_0 = 3$ hours

Start 0900

Finish 1200

He would observe 1200 on his clock because time is only dilated relative to the Earth observer

13. $\ell_0 = 600$ m

$\upsilon_0 = 0.80c$

a. they are both inertial frames provided they are moving at a constnt velocity.

b. $t = \frac{s}{v} = \frac{600}{3x10^8}$

= 2.00 x 10^{-6} s

c. distance laser travels = length of fighter as seen from mothership plus distance spaceship travels in that time

$$c\Delta t = \gamma\ell_0 + \upsilon\Delta t$$

$$\Delta t = \frac{\gamma\ell_0}{c-\upsilon} = \frac{600\sqrt{1-0.64^2}}{c-0.8c}$$

$$= \frac{461.0}{0.2 \text{ x } 3x10^8}$$

= 6 x 10^6 s

14.

15. $\upsilon_p = 3.5 \text{ x } 330 \text{ ms}^{-1}$ = 1155 ms^{-1}

$\upsilon_m = 700 \text{ ms}^{-1}$

a. $\upsilon_{missile} = 1155 + 700 = 1855 \text{ ms}^{-1}$

b. $u = \frac{v-u'}{1+\frac{vu'}{c^2}}$

$$= \frac{1855}{1+\frac{1155x700}{(3x10^8)^2}}$$

$$= \frac{1855}{\square 1}$$

=1855 ms^{-1}

The same because not relativistic

16. $\upsilon = 0.25c$

$u' = 0.65\ c$

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

$$= \frac{0.90c}{1 + 0.25x0.65}$$

$$= \frac{0.9c}{1.625}$$

$=0.554c\ ms^{-1}$

17. $m_0 = 3400\ kg$

$\upsilon = 2.25x10^8\ ms^{-1}$

$m = \gamma m_0$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{3400}{\sqrt{1 - (\frac{2.25x10^8}{3x10^8})^2}}$$

$$= \frac{3400}{0.6614}$$

$=5.14\ x\ 10^3\ kg$

18. $\ell_0 = 158\ m$

$\upsilon = 0.925c$

$m_o = 8.00\ x\ 10^5\ kg$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{8.00x10^5}{\sqrt{1 - 0.925^2}}$$

$= 2.11\ x\ 10^8\ kg$

$$\ell = \ell_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$= 158\sqrt{1 - 0.925^2}$$

$= 60\ m$

19. $m=3m_o$

$m = \gamma m_0$

$$m_0 = \frac{m}{\gamma}$$

$$m_0 = 3m_0 \sqrt{1 - \frac{\upsilon^2}{c^2}}$$

$$\sqrt{1 - \frac{\upsilon^2}{c^2}} = \frac{1}{3}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{\upsilon^2}{c^2} = \frac{8}{9}$$

$$\upsilon^2 = \frac{8}{9}c^2$$

$\upsilon = 0.943c$

Problem Set 15.1: The standard model

1. The hundreds of known particles are all made from: 6 quarks, 6 leptons, 6 antiquarks, 6 antileptons, and the force carriers.

2. Neutrons and protons are made up of quarks, which are held together by gluons
   Electrons are fundamental particles and are classified as leptons.

3. Baryons and mesons are hadrons.

   BARYONS - are any hadron made of three quarks (qqq). Protons and neutrons are baryons because they are each made of three quarks – protons two up and one down quark (uud) and neutrons one up and two down (udd).

   MESONS are hadrons made from a quark and its anti-quark (eg pion or pi-meson). One example of a meson is a pion (+), which is made of an up quark and a down anitiquark. The antiparticle of a meson just has its quark and antiquark switched, so an antipion (-) is made of a down quark and an up antiquark. Because a meson consists of a particle and an antiparticle, it is very unstable. The K meson lives much longer than most mesons, which is why it was called "strange" and gave this name to the strange quark, one of its components.

4. Both muon and antimuon have a mass of 105.66 MeV/c^2. (Assuming the particles are slow moving)

   (a) The two photons will each have energies of 105.66MeV.

   (b) Using $E = h\nu = hc/\lambda$

   $\lambda = hc/E = 6.62 \times 10^{-34} \times 3\times10^{8}/(105.66 \times 10^{6} \times 1.6\times10^{-19}) = 4.2 \times 10^{-15}$ m

   (c) Two photons are required to conserve momentum

   (d) They must travel in opposite directions to conserve momentum

   (e) The photons are in the gamma radiation part of the e/m spectrum.

5. (a) n → p + e^- + ______

	CHARGE	BARYON No.	LEPTON No.
LHS	0	1	0
RHS	0	1	1
Balance	0	0	-1

An anti-neutrino is required on the RHS to balance

(b) ___ + n → ____ + e^-

	CHARGE	BARYON No.	LEPTON No.
LHS	0	1	0
RHS	-1	0	1
Balance	1	-1	-1

electron antineutrino(LHS) and proton (RHS)

(c) π^+ → μ^+ + ____

	CHARGE	BARYON No.	LEPTON No.
LHS	1	0	0

RHS	1	0	1
Balance	0	0	-1

A muon antineutrino is given off.

(d) p → n + ν_e + ____

	CHARGE	BARYON No.	LEPTON No.
LHS	0	1	1
RHS	1	1	0
Balance	1	0	-1

An antimuon

6. Each γ–ray must have energy of 511keV (the rest energy of an electron and positron). To conserve momentum two γ–rays are produced travelling in opposite directions.

7. Positrons will travel anti-clockwise

8. Yes an object can accelerate while keeping the same speed, it they are undergoing circular motion at constant angular velocity.

9. Conservation of momentum is violated – the particles have momentum in the y-direction (toward top of page) which they didn't possess before the collision.

10. (a) Friction is caused by residual electromagnetic interactions between the atoms of the two materials. The force carriers are photons and W and Z bosons.

    (b) Nuclear bonding is caused by residual strong interactions between the various parts of the nucleus. The force carriers are gluons.

    (c) The planets orbits due to gravitons.

11. (a) Weak and Gravity interactions act on neutrinos

    (b) Weak (W+, W-, and Z) interactions have heavy carriers

    (c) All of interactions act on the protons in you

12. Gluons cannot be isolated because they carry colour charge themselves.

13. Gravitons are hypothetical particles to explain the 'force' of gravity. They have not been observed. (Gluons have been observed indirectly.)

Problem Set 15.2: General revision questions

1. Using $\lambda = \dfrac{hc}{pc}$

   Where $pc = \sqrt{2.E_K.m_0c^2}$

   Here E_K = 7.0 x 10^{12} eV
   pc = 1.1469x10^{11} eV
   m_0c^2 = 511 x 10^3 eV
   hc = 1239.84 eV.nm (this is a constant)
   □ = 1.08 x 10^{-17} m

2. Using $\dfrac{v}{c} \approx 1 - \tfrac{1}{2}\left(\dfrac{m_0c^2}{E_{tot}}\right)^2$ for v ≈ c

   Where $E_{tot} \approx E_K$ = 3.00 x 10^9 eV and m_0c^2 = 5.11 x 10^5 eV
   v = 0.999999985493c

3. Using m_{Rel} = □.m_0 where *g = 707.1 and* m_0 *= 1.67 x* 10^{-27} *kg*
   m_{Rel} = 1.18 x 10^{-24} kg.

4. E = 7.53 x 10^{-13} J
   $m = E/c^2$
   Mass = 8.38 x 10^{-30} kg

5. (a). m_ec^2 = 8.19 x 10^{-14} J, 5.11 x 10^5 eV
   (b).□m_ec^2 = 3.12x 10^{-13}J, 1.95 x 10^6 eV
   (c). E_K = (□-1) m_ec^2 = 2.303 x 10^{-13} J, 1.44 x 10^6 eV

6. Using: $\dfrac{v}{c} \approx \sqrt{1 - \left(\dfrac{m_0c^2}{E_{tot}}\right)^2}$

   $E_{tot} = E_k + m_ec^2$
   E_K = 40,000 eV
   (a). v_{max} = 0.374c
   (b). 40,000 eV

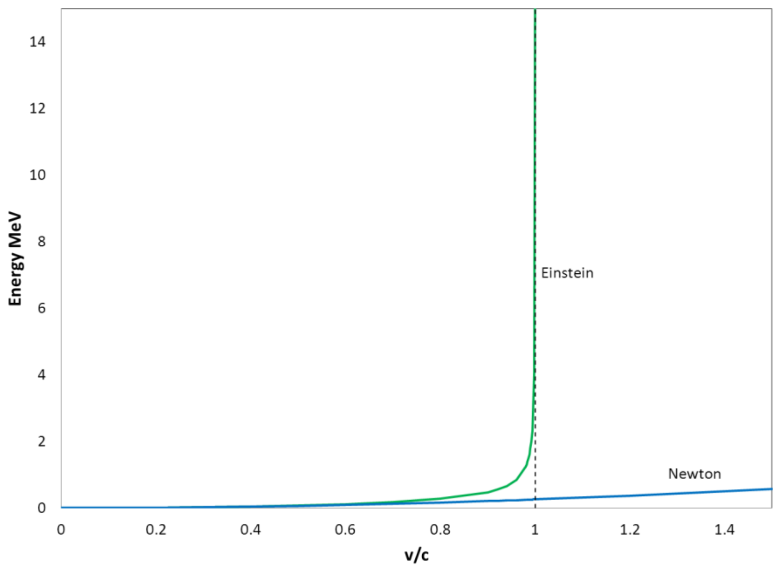
7. No energy is released – 605 MeV is required to make this reaction occur:
   (139.6 + 938.3) - (1189.4 + 493.7) = -605MeV

8. Mass Pa236 = 236.04868 u
   Mass U236 = 236.045568 u
   Mass difference = 0.003112 u
   □mc^2 = = 2.9 MeV
   KE of recoil nucleus = approx. 33eV which is negligible
   E beta = 2.9 MeV
   V beta = 0.989c

9. □ = 707.1 m_{rel} = 1.18 x10^{-24} kg

10. 2.16 x 10^{-23} kg 0.00209 km

11. In the Synchrotron electrons are accelerated to velocities approaching the velocity of light. The graphs of both the non-relativistic energy and the relativistic energy are shown



12. (a). 0.511 MeV
    (b). p_{rel} = 2.05 x 10^{-14} kg.m/s' $p_{classical}$ = 2.73 x10^{-22} kg.m/s
    (c). 6.15 x10^{-06} J, 3.84 x10^{13} eV

13. See problem 2.

14. It is moving away (red shifted)

    Using: $\frac{v}{c} = \frac{\left(\frac{\lambda_0}{\lambda}\right)^2 - 1}{\left(\frac{\lambda_0}{\lambda}\right)^2 + 1}$

    v = 0.72c (moving apart)

15. Wavelength green light = 540nm (±30nm)
    Wavelength red light = 700nm (±30nm)
    v = 0.25c toward the light

16. Relativistic mass =2.00 x10^{-30} kg
    KE: 9.77 x10^{-14} J, 6.10 x10^{5} eV

17. Gamma x 26 □s = 3.2 x 26 □s = 83 □s

19. See 18 – note here the half-life is stated at 260 □s not 26 □s.
    (a) They will appear to have a half-life of 830 ms.
    (b) distance travelled = 0.95 x c m/s x 830 x 10^{-9} s = 236m
    (c). 74m

20. (a). Toward the Earth
    (b). 0.2c
    (c). Apart
    (d) 0.24c